Chapter 16

M2: Advection of Sharp Salinity Plume

Contents

16.1 Problem Specification

M2 Advection of sharp salinity plume in uniform flow.

Focus advection algorithm, numerical dispersion.

Channel geometry and hydrodynamic initial and boundary conditions are the same as schematic application M1. Also, use same fixed computational time step $\Delta t = 120$ s and fixed computational space step of $\Delta x = 500$ ft. The longitudinal dispersion coefficient $E_x = 0$.

The contaminant initial conditions at $t = 0$ are

$$C(x, 0) = 1 \exp\left[-c\left(\frac{x - x_0}{b_{\mathrm{M2}}}\right)^2\right] \quad \text{ft} \tag{16.1.1}$$

where $c = \ln 2 = 0.6931$, $x_0 = 5{,}000$ ft and $b_{\mathrm{M2}} = 250$ ft.

The contaminant open boundary conditions are no contaminant inflow and unconstrained contaminant outflow.

Compute and write to file in the STANDARD FORMAT the initial conditions at $t = 0$ and the model predictions for every time step to $t = 30\Delta t$ s.

16.2 Background

The numerical dispersion and solution oscillation problems become much more prevalent in the modeling of sharp contaminant fronts. The sharper plume was intended to push this envelope.

The Equation 15.2.3 analytical solution remains appropriate, but the initial conditions have changed to Equation 16.1.1. The analytical solution is shown in Figure 15.1b. This is very much steeper, and a very challenging test of an advection code.

16.3 Contra Costa Water District

No response.

16.4 Department of Water Resources

Figure 16.1a shows the DWR-predicted[1] evolution of the salinity plume. Figure 16.1b shows the error field, computed as $C_{\text{numerical}}(x,t) - C_{\text{analytical}}(x,t)$. Ideally, part (a) should be identical to Figure 15.1a and part (b) should be zero throughout. The Figure 16.1a numerical model prediction is not identical to the Figure 15.1a analytical solution. As for M2, the differences are a direct consequence of the DWR re-definition of problem. The plume is advected in the positive x direction in Figure 16.1a, but in the negative x direction in 15.1a. In addition, data was reported only every 900 s, rather than 120 s, so that the data set is very sparse in the t direction. The actual DWR predictions are indicated by the circle markers. The balance of the surface plot is the standard response of surface or contouring, which interpret the sparse t data as isolated ridges. The circle markers suggest excellent algorithm performance. This is confirmed by the error field plot in Figure 16.1b where the maximum error is of order 0.005 and insignificant. The expected excellent performance of the DWR model for pure advection has again been demonstrated, despite the purposefully sparse spatial resolution of the initial plume imposed in the M2 problem specification.

Figure 16.1c shows the DWR-predicted contaminant mass balance. Unfortunately, the poor t resolution makes it difficult to be certain if this or is not an acceptable result.

[1]The DWR data file has the predicted solution reported only every 900s, instead of the required 120 s, the x axis directed in the wrong direction, and the time and reach number listed in inverted order. Appropriate corrections have been made for the following analyses.

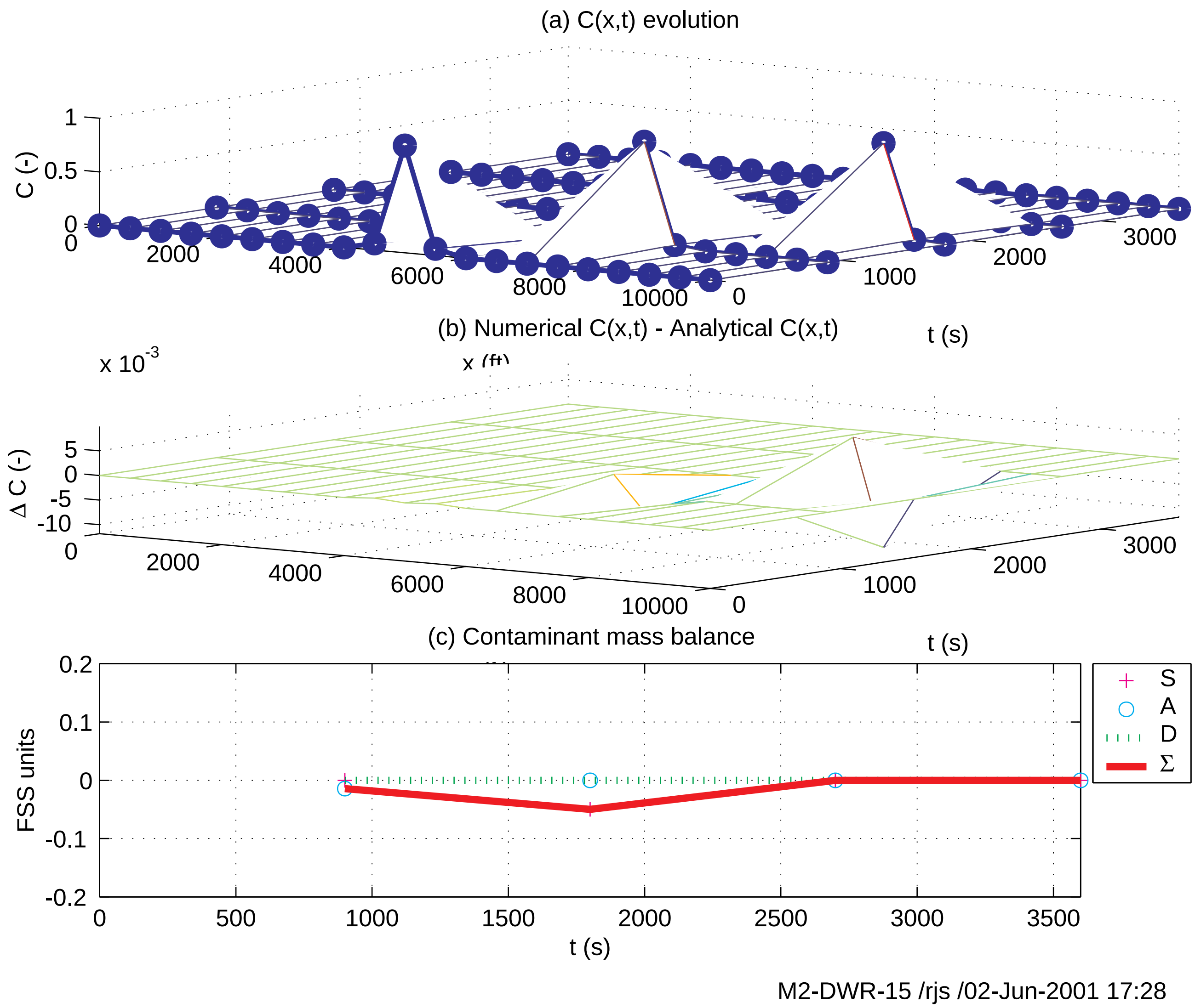

Figure 16.1: M2 DWR-predicted Advection of Salinity Plume, Error Field and Contaminant Mass Balance.

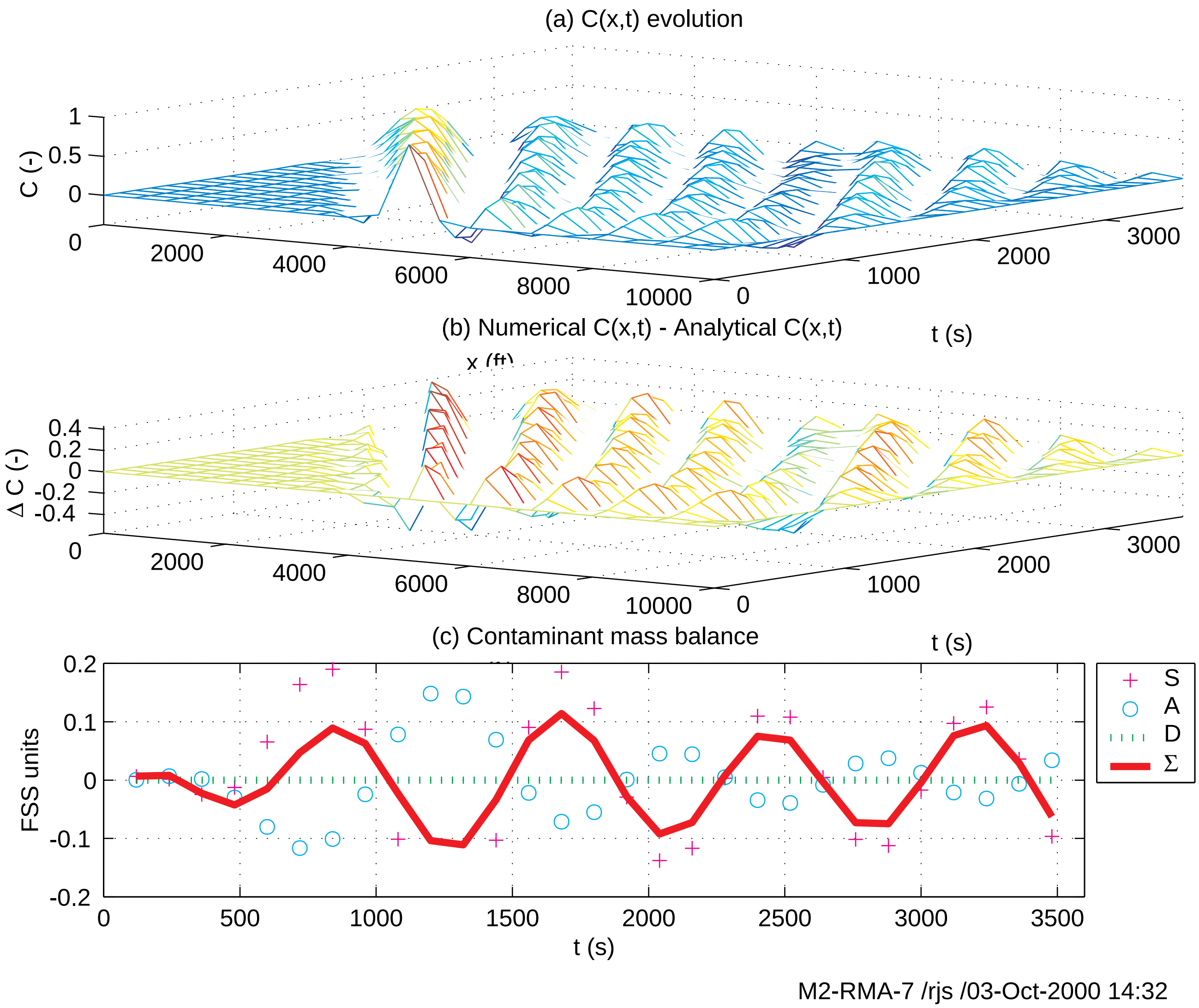

Figure 16.2: M2 RMA-predicted Advection of Salinity Plume, Error Field and Contaminant Mass Balance.

16.5 Resource Management Associates

Figure 16.2a shows the RMA-predicted evolution of the salinity plume. Figure 16.2b shows the error field, computed as $C_{\text{numerical}}(x,t) - C_{\text{analytical}}(x,t)$. Ideally, part (a) should be identical to Figure 15.1a and part (b) should be zero throughout. Overall, this is the expected result. Figure 16.2a vaguely follows Figure 15.1a, but shows advanced signs of terminal numerical dispersion and solution oscillation illness. The error field, Figure 16.2b, shows severe numerical dispersion and solution oscillations, more than sufficient to destroy the integrity of the predicted solution, as any experienced modeler knows. This IS the expected result; it is a case of stretching a model beyond its limits of validity.

Figure 16.2c shows the RMA-predicted contaminant mass balance. Contaminant mass is not conserved. This is a direct consequence of rapidly-varied changes that accompany the solution oscillations.