Chapter 6

H3: Transition to Steady State Tidal Circulation

Contents

6.1 Problem Specification

H3 Transition from quiescent initial conditions to steady state circulation in uniform channel with upstream fresh water discharge and downstream tide.

Focus initial transients, mixed boundary conditions.

Channel bed slopes linearly upwards from downstream point F to upstream point L. The trapezoidal channel bed width B is 50 ft. At F, $x_F = 0$ ft, $Z_F =$ -20.00 ft and

$$\eta(x_F, t) = 3 \sin \omega t \text{ for } t > 0 \tag{6.1.1}$$

where $\omega = 2\pi/T$, the tidal period T being 12.5 hours.

At L, $x_L = 250{,}000$ ft, $Z_L = +10.00$ ft and

$$Q(x_L, t) = -1,000 \text{ ft}^3/\text{s for } t > 0 \tag{6.1.2}$$

Channel friction factor is constant at Darcy-Weisbach $f = 0.03$ or Manning $n = 0.02$.
Initial conditions at $t = 0$ are quiescence.

$$\eta(x, 0) = +20 \quad \text{ft}, \qquad Q(x, 0) = 0 \tag{6.1.3}$$

Use a fixed computational space step $\Delta x = 500$ ft and a fixed computational time step $\Delta t = 30$ s.

Compute and write to file in the STANDARD FORMAT the initial conditions at $t = 0$ and the model predictions for every time step to $t = 2T$.

6.2 Background

Unsteady flow in a estuarine channel is an initial boundary value problem. It is driven by the initial conditions and by the open boundary conditions. This problem was intended to investigate initial start-up transients, and their interaction with both η and Q open boundary conditions.

In a hyperbolic system, disturbances or waves are created wherever and whenever there is a change in the forcing. There are potential sources at every computational node and at every time:

- If the prescribed initial conditions do not satisfy both mass and momentum conservation locally and instantaneously, this out of balance will force a disturbance at that node, which will propagate throughout the solution field. This possibility is avoided in the present problem by assigning initial conditions that must satisfy mass and momentum conservation throughout, namely hydrostatic equilibrium.

- If at any later time the local conditions do not satisfy both mass and momentum conservation, this out of balance will also force a disturbance at that node, which will also propagate throughout the solution field. If the code is correct, this will not happen. If there are coding errors, this will certainly happen. This possibility was investigated in Problem H1.

Any dynamical system has a *forced mode* response, the response to sustained forcing, and a *free mode* response, the response to sudden but unsustained forcing. Our physical interest is centered on the forced mode response, but free mode responses are also driven by the sudden discontinuity in the upstream and downstream boundary forcing at $t = 0$.

Free mode responses in estuarine channels (Bode and Sobey 1984) have predictable periods. Free mode amplitudes are more difficult to estimate, but the free mode response does decay exponentially with channel friction at a predictable rate. This exponential decay gives rise to the *initial transient* designation, and the familiar need for a spin up time or warm up time for long wave models. For the present problem, the initial transient amplitudes unfortunately happen to be too small to identify.

In the present problem also, disturbances are created only at the boundaries, so that it is possible to follow the evolution of these disturbances, which is to follow the response to the specific boundary forcing.

Boundary forcing is typically

- $\eta(x_B, t)$ is specified, or
- $Q(x_B, t)$ is specified, or
- some relationship $f\left[\eta(x_B, t), Q(x_B, t)\right]$ is specified.

where x_B is the boundary location. The common open boundary condition are η and Q conditions. The coding arrangements for η and Q conditions are different, as are the coding arrangements for upstream and downstream locations.

The present problem specifies $\eta(x_F, t)$ and $Q(x_L, t)$. The downstream boundary specifies an ocean tide, which will force the familiar flood and ebb tide response in the lower estuary. The upstream boundary represents a constant fresh water inflow, which will dominate flow in the upper reaches and inhibit deep flood tide penetration into the solution field.

In the mass and momentum conservation balances here, the opportunity can be taken to investigate the "steady state" response to tidal forcing by focusing on the second tidal cycle at times $T \leq t \leq 2T$. The conservation balances were evaluated every computation time step, but for plot clarity, need only be presented every 15 minutes. Figure 6.1 from the ESTFLOW model shows what is to be expected. The first notable feature is the smoothly varying but cyclic evolution of all terms. The momentum balance in particular shows the evolving balance among inertia, gravity and friction. The nonlinearity is also seen in the temporal asymmetry of these time histories.

One feature of this problem, the sudden drawdown at F from an initial condition of $\eta(x_F, 0)$ = +20 ft to a tide of $\eta(x_F, t) = 3\sin\omega t$ for $t > 0$, has drawn some negative comment[1]. The intention was to achieve very clean initial conditions (quiescence), so that the propagation of the sudden changes at the boundaries could be identified. This it did very successfully, and without any difficulty in all three codes. The supplementary intention, a focuss on the steady-state tidal balances as early as the second complete tidal cycle was also achieved without difficulty.

[1]The following commentary was provided by DWR (2 February 2001): "This problem specifies a sudden drop of 20 ft at the boundary, followed by a tidal forcing at the boundary. This is similar to a dam-break problem. Suitability of this test problem to Sacramento-San Joaquin Delta applications is highly questionable."

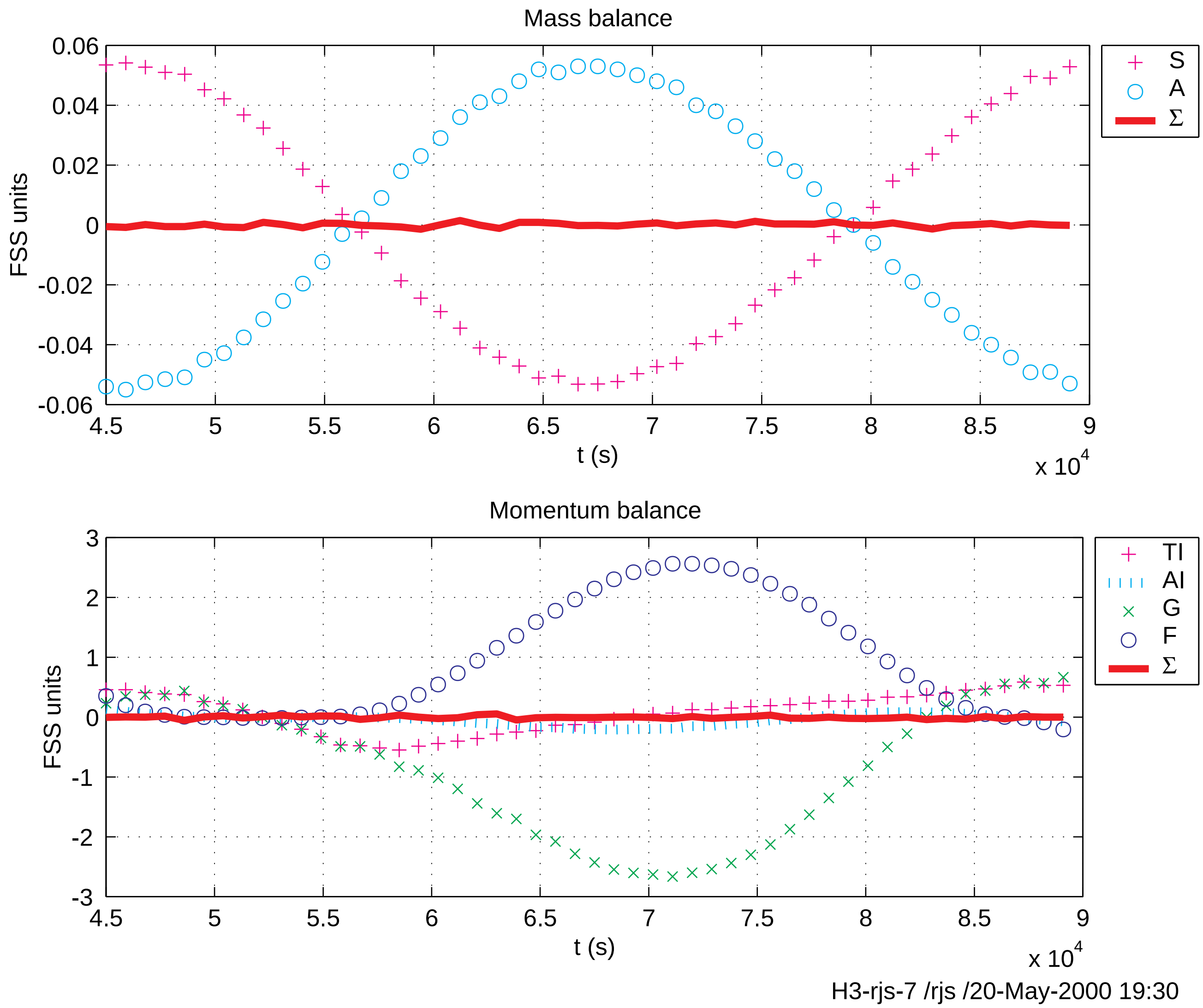

Figure 6.1: H3 ESTFLOW-predicted conservation balances at $x = 3{,}000$ ft.

6.3 Contra Costa Water District

The CCW-predicted η solution field evolution is shown in Figure 6.2. Part (a) shows the entire field as a contour plot. Following the $\eta = \pm 2$ ft contours shows the expected flood and ebb tide penetration. Parts (b) and (c) shown surface plots of the detail at the downstream and upstream ends respectively. In both cases, the expected propagation of the sudden change in the boundary conditions along an incoming characteristic path is clearly seen. There is no evidence of free mode response.

The CCW-predicted Q solution field evolution is shown in Figure 6.3. Parts (a) through (c) show equivalent detail to the η plot. The flood tide penetration is somewhat less obvious than in Figure 6.2a, because of the fresh water throughflow, but can nonetheless be identified through the time history of say the -2000 ft^3/s contour. Parts (b) and (c) again show the expected propagation of the sudden change in the boundary conditions along an incoming characteristic path.

Figure 6.4 shows the time history of the mass and momentum balances at location $x = 3{,}000$ ft for the second tide cycle. The step-like irregularity in these plots strongly hints at a coding problem[2]. Both mass and momentum are approximately conserved at the selected location, though the step-like irregularity is also seen in the Σ terms. There is a hint of this irregularity also in Figure 6.3, where the contours are somewhat less smooth on the flood tide than expected.

[2]The following commentary was provided by CCW (Shum, 27 April 2001): “The ”step-like” irregularities are a direct consequence of the discretized input of stage at the downstream boundary in FDM. This input is at hourly intervals with two decimal-place accuracy. FDM uses linear interpolation of hourly inputs for intermediate time-steps within each hour. The temporal gradient of stage at the downstream boundary is therefore piecewise continuous. . . . There are a number of ways to address this ”coding problem”, but the relevance of these ”fixes” to Delta simulations is questionable.”

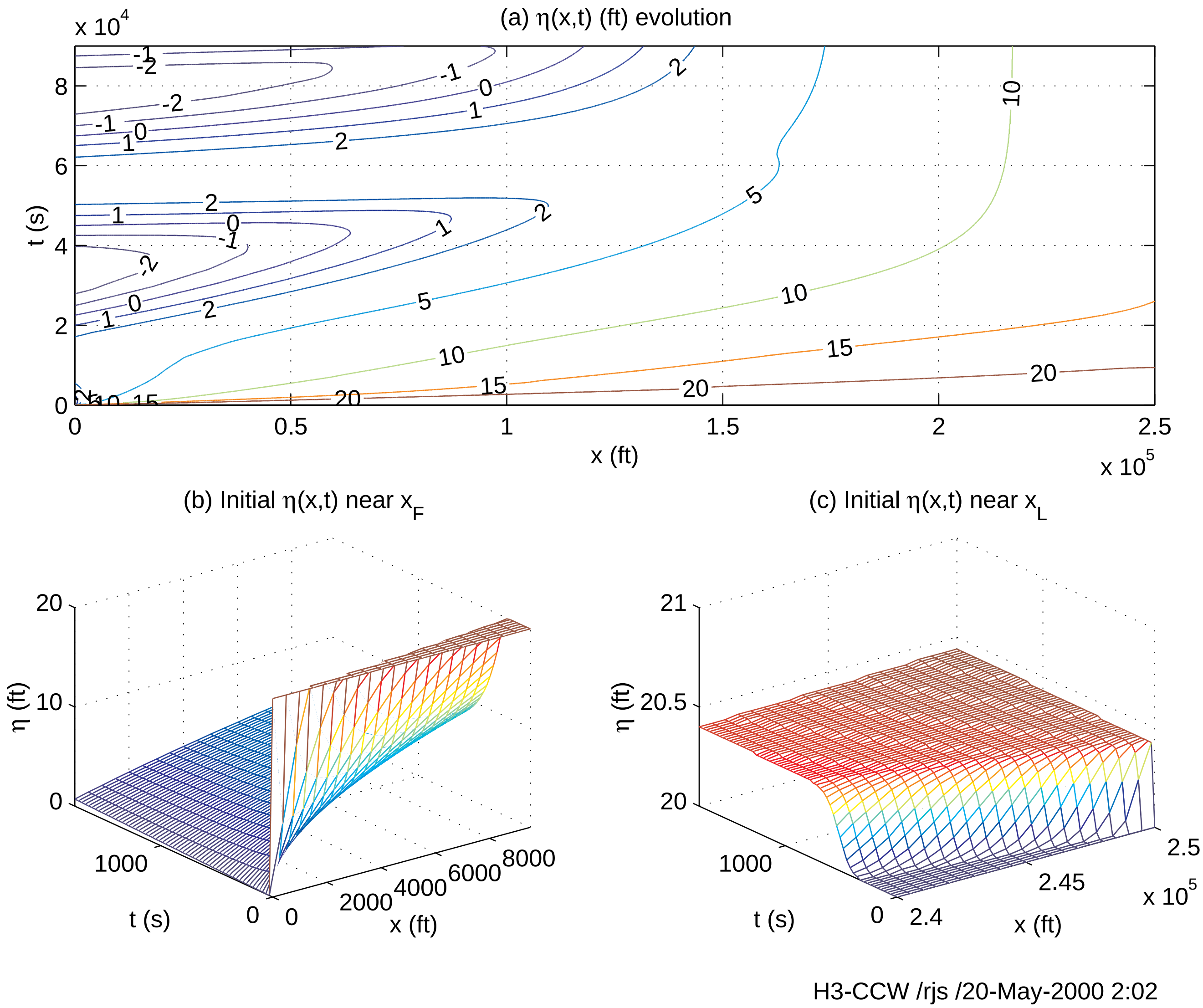

Figure 6.2: H3 CCW-predicted η solution field evolution.

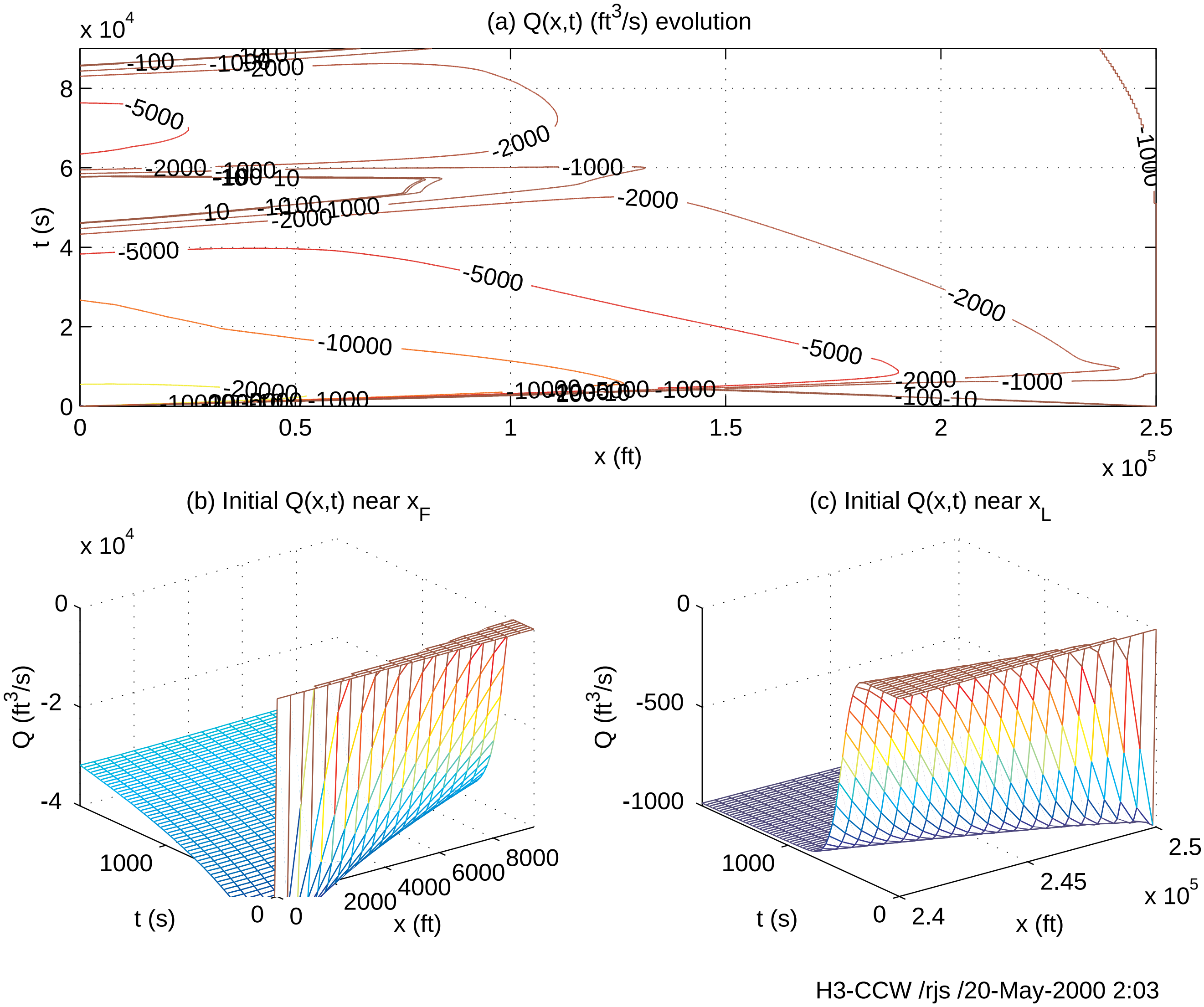

Figure 6.3: H3 CCW-predicted Q solution field evolution.

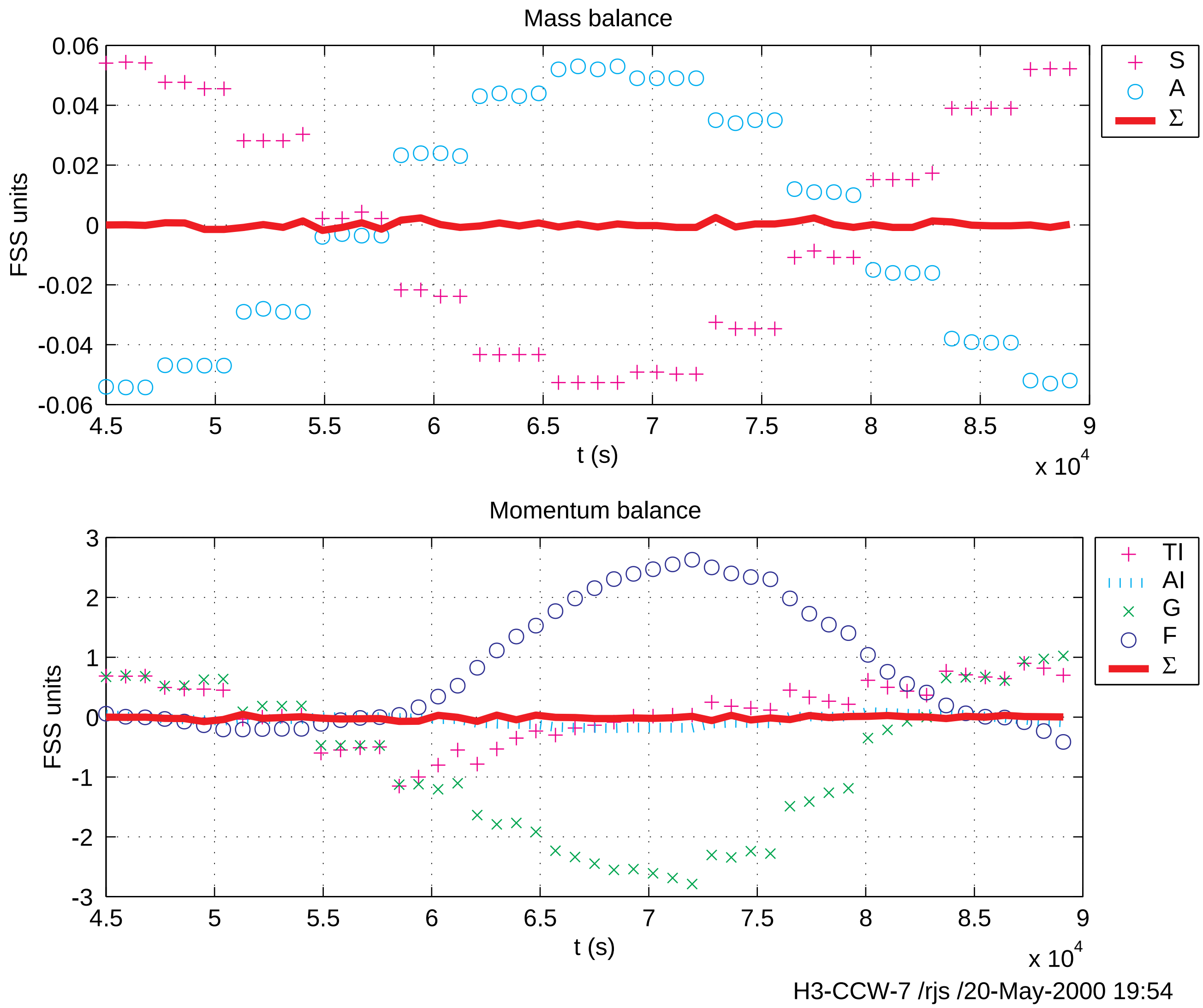

Figure 6.4: H3 CCW-predicted conservation balances at $x = 3{,}000$ ft.

6.4 Department of Water Resources

The DWR-predicted[3] η solution field evolution is shown in Figure 6.5. Part (a) shows the entire field as a contour plot. Following the $\eta = \pm 2$ ft contours shows the expected flood and ebb tide penetration. Parts (b) and (c) shown surface plots of the detail at the downstream and upstream ends respectively. In both cases, the expected propagation of the sudden change in the boundary conditions along an incoming characteristic path is clearly seen. There is no evidence of free mode response.

The DWR-predicted Q solution field evolution is shown in Figure 6.6. Parts (a) through (c) show equivalent detail to the η plot. The flood tide penetration is somewhat less obvious than in Figure 6.5a, because of the fresh water throughflow, but can nonetheless be identified through the time history of say the -2000 ft^3/s contour. Parts (b) and (c) again show the expected propagation of the sudden change in the boundary conditions along an incoming characteristic path. There is a hint of initial transients in the Q response in part (c); these propagate from the tidal boundary but rapidly decay to friction in the expected manner.

Figure 6.7 shows the time history of the mass and momentum balances at location $x = 3{,}000$ ft for the second tide cycle. Both mass and momentum[4] are conserved at the selected location.

[3] The DWR data file incorrectly reports the computational time step Δt as 1 s; it was apparently not the specified 30 s, but 60 s. The time step has been changed to 60 s for the following analyses. The x axis is also reversed.

[4] The October 1999 version of these DWR-predictions showed a major imbalance in momentum. The problem was eventually traced to a coding error, which necessitated the March 2000 revisions. Without independent benchmark evaluations, this potentially serious problem may not have been identified.

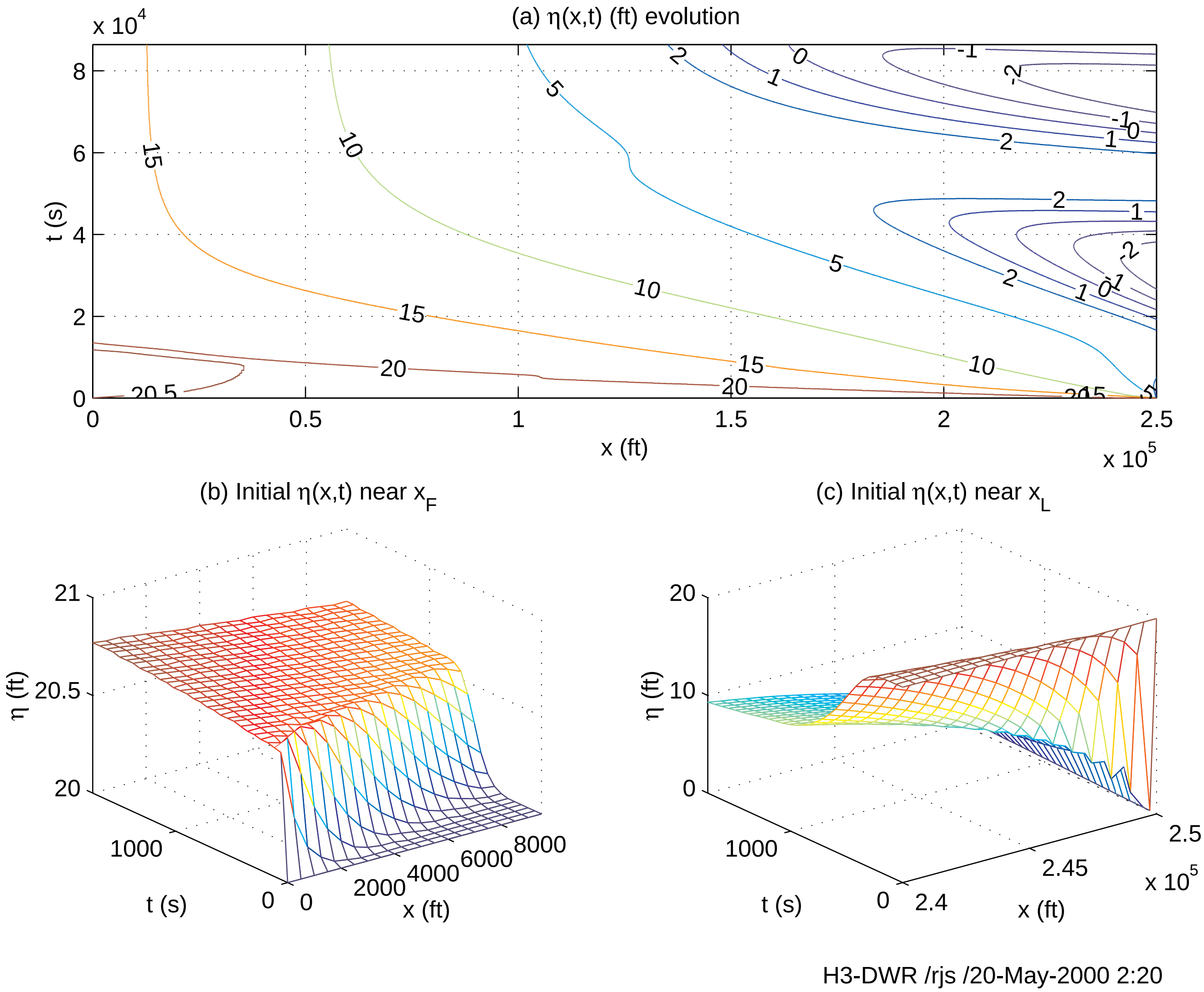

Figure 6.5: H3 DWR-predicted η solution field evolution.

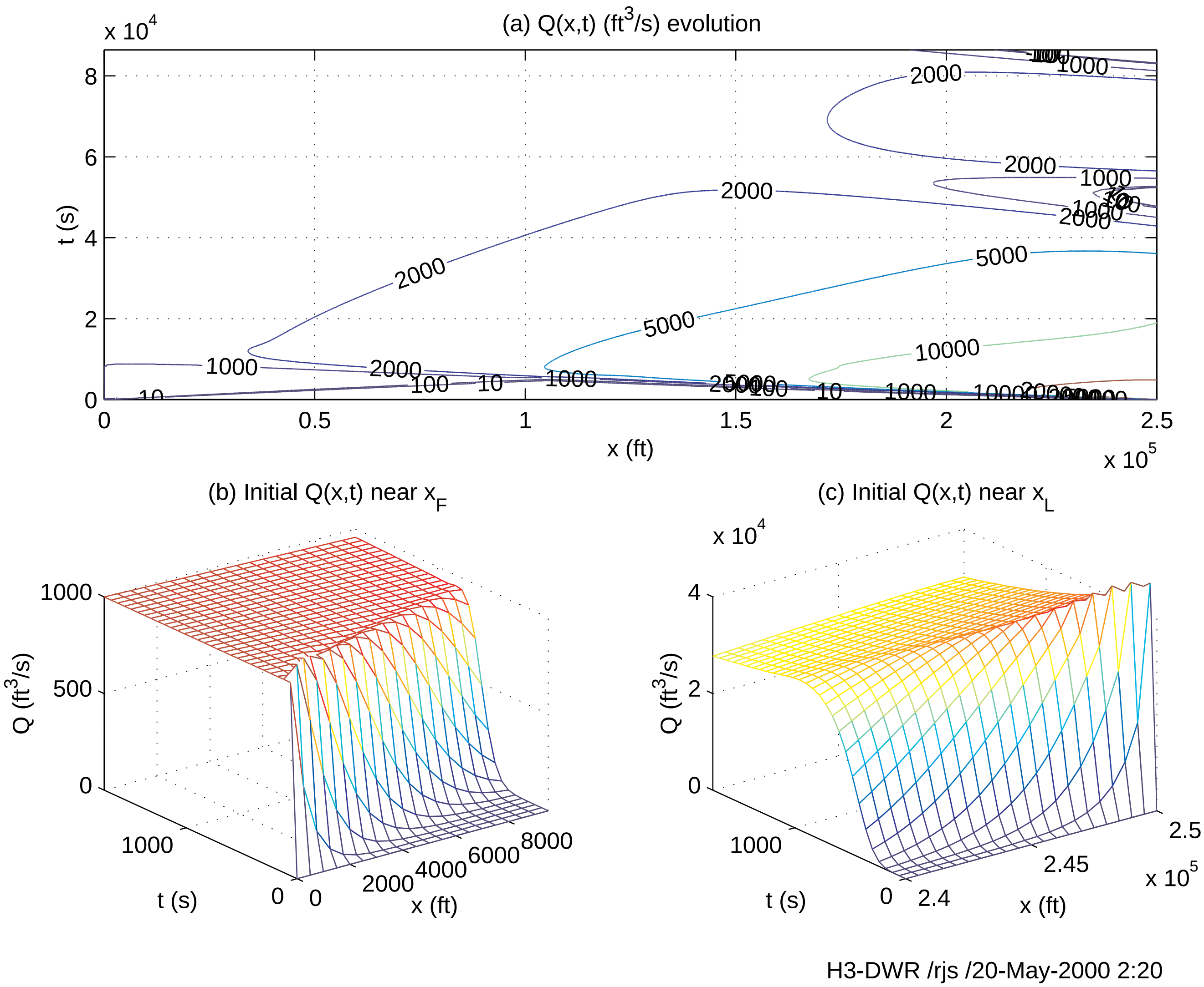

Figure 6.6: H3 DWR-predicted Q solution field evolution.

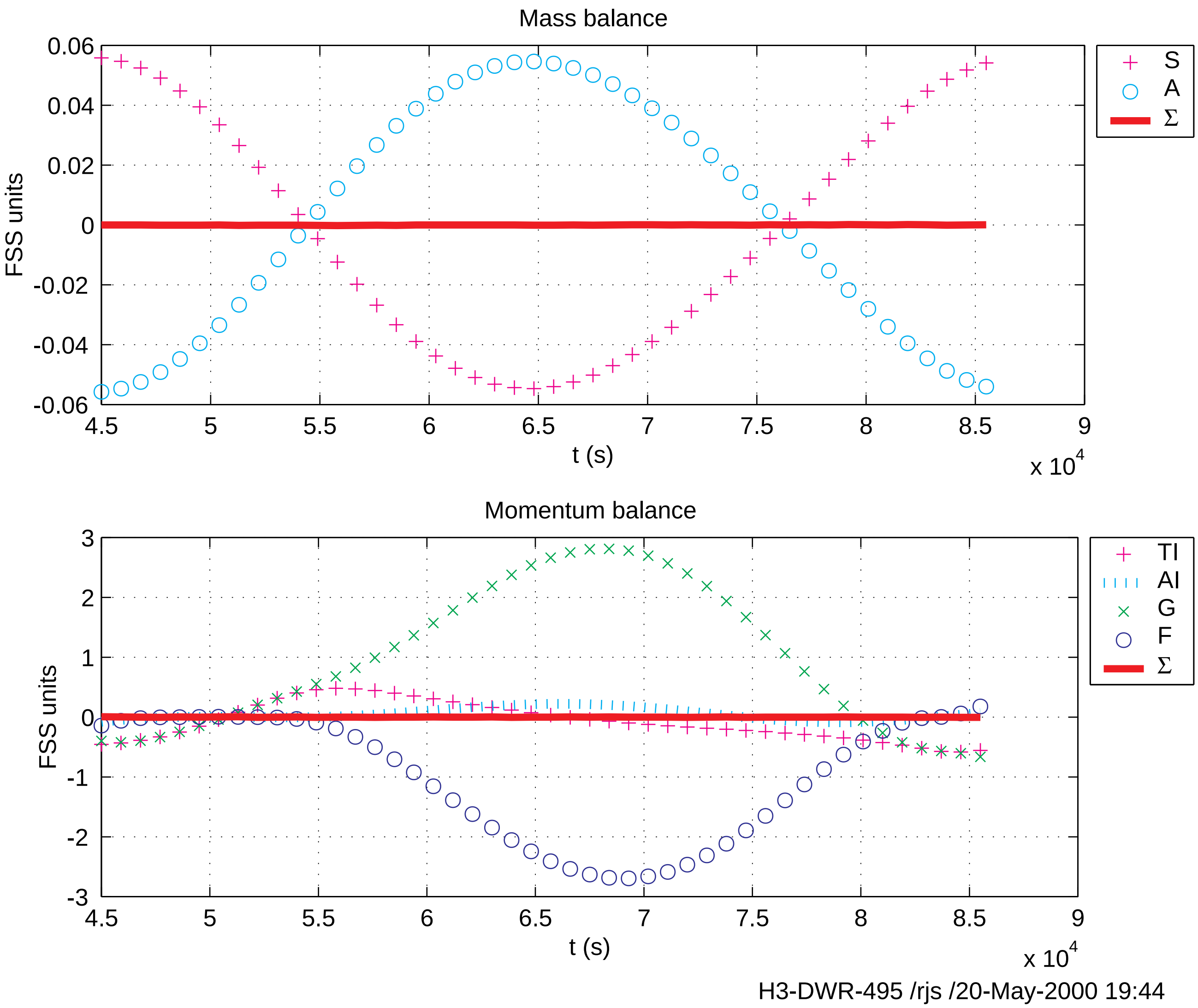

Figure 6.7: H3 DWR-predicted conservation balances at $x = 247{,}000$ ft.

6.5 Resource Management Associates

The RMA-predicted η solution field evolution is shown in Figure 6.8. Part (a) shows the entire field as a contour plot. Following the $\eta = \pm 2$ ft contours shows the expected flood and ebb tide penetration. Parts (b) and (c) again show the expected propagation of the sudden change in the boundary conditions along an incoming characteristic path. Part (c) shows initial transient response, propagating from the upstream flow boundary. These eventually decay to friction in the expected manner.

The RMA-predicted Q solution field evolution is shown in Figure 6.9. Parts (a) through (c) show equivalent detail to the η plot. The flood tide penetration is somewhat less obvious than in Figure 6.8a, because of the fresh water throughflow, but can nonetheless be identified through the time history of say the -2000 ft^3/s contour. Part (b) again shows the expected propagation of the sudden change in the boundary conditions along an incoming characteristic path, though there are hints of Q solution irregularity that was not apparent in the η trace, Figure 6.8b. This is perhaps reminiscent of the Q but not η response irregularity in Figure 5.10. Part (c) also shows initial transient activity, but this was mirrored in the η trace. These initial transients again decay to friction in the expected manner.

Figure 6.10 shows the time history of the mass and momentum balances at location $x = 3{,}000$ ft for the second tide cycle. Both mass and momentum are conserved at the selected location, and there is no suggestion of any difficulty.

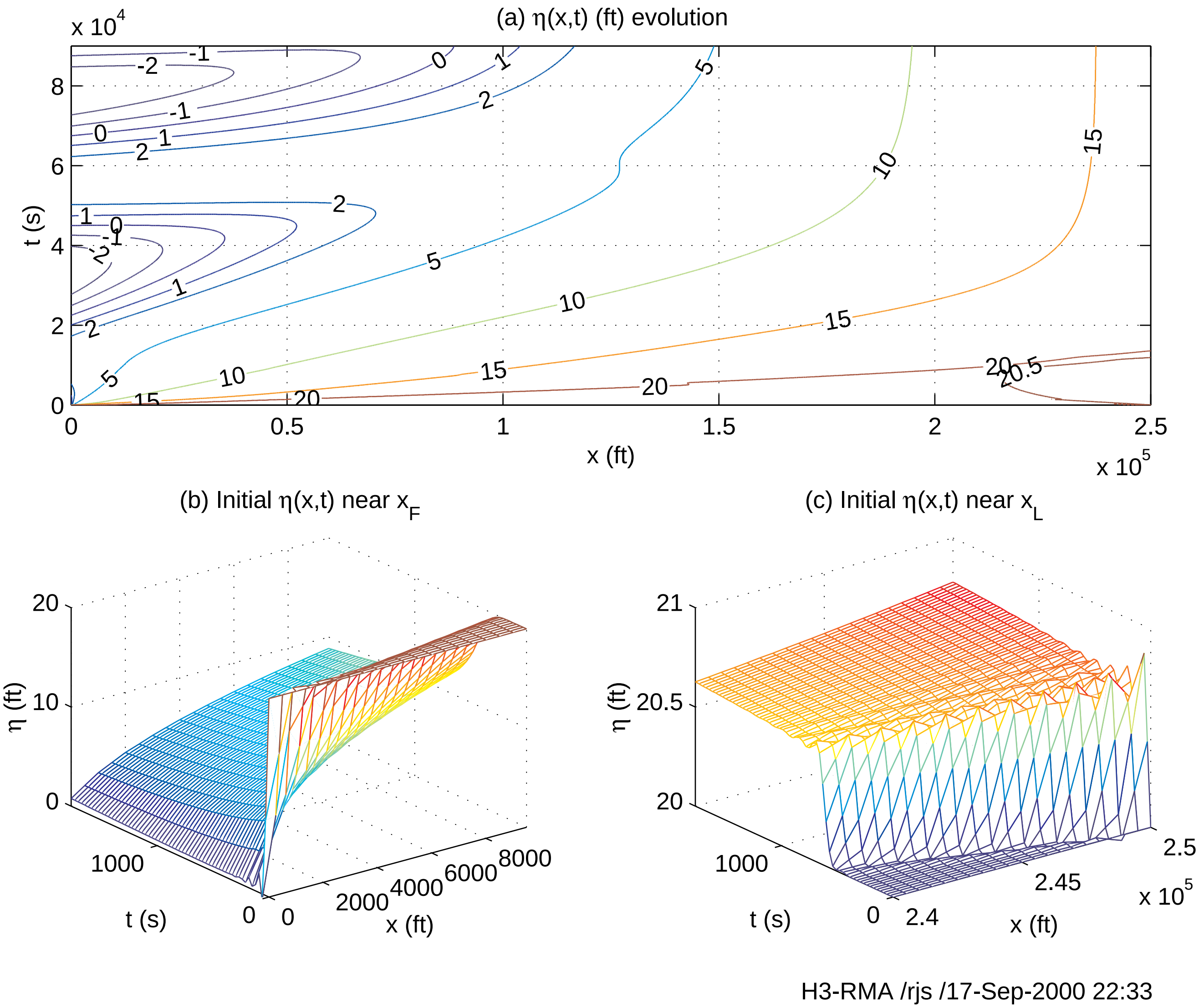

Figure 6.8: H3 RMA-predicted η solution field evolution.

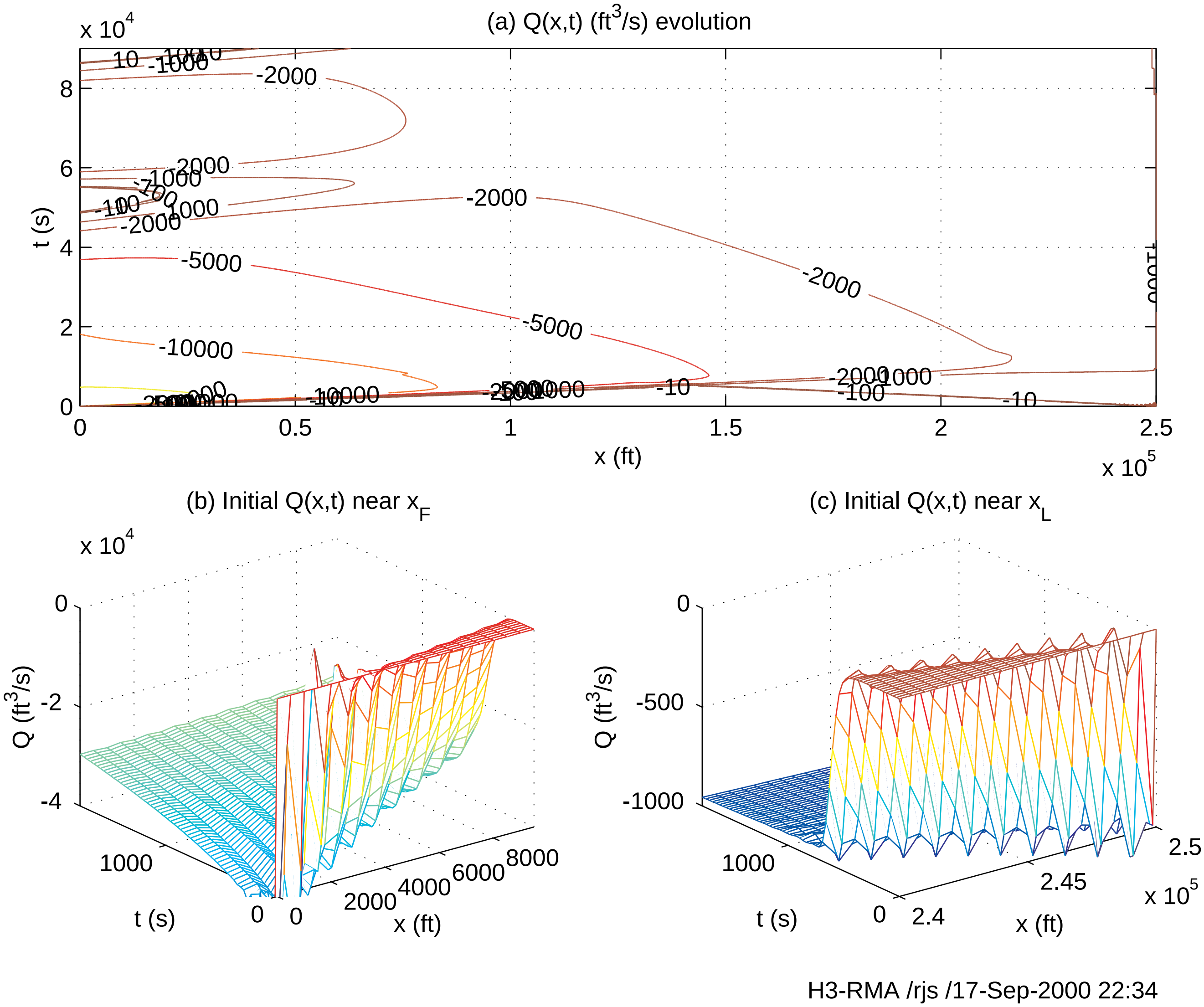

Figure 6.9: H3 RMA-predicted Q solution field evolution.

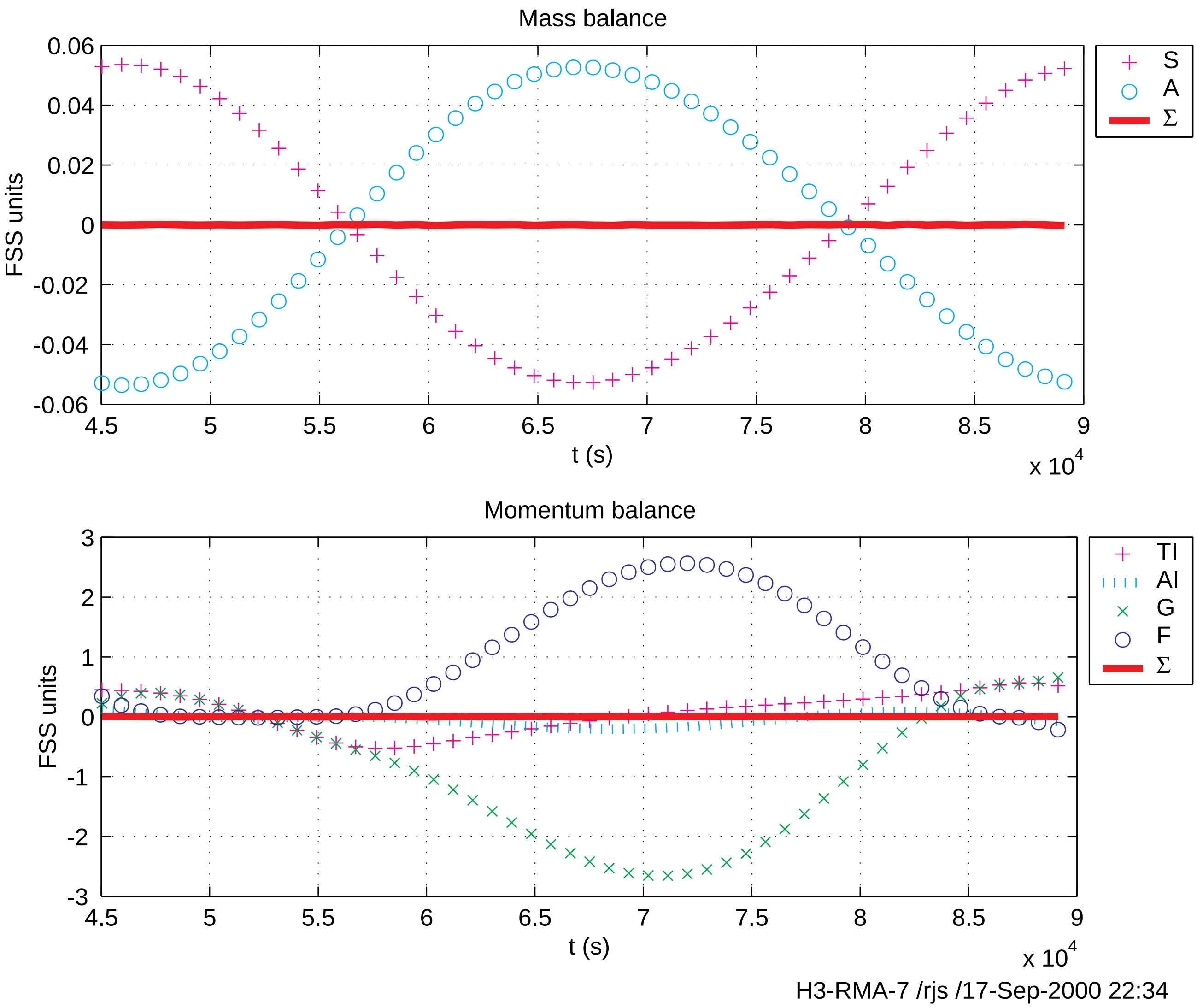

Figure 6.10: H3 RMA-predicted conservation balances at $x = 3{,}000$ ft.

6.6 Response Comparisons

As a final comment, the evolution of the DWR and RMA solutions should be identical. They have the same channel geometry, the same initial conditions and the same boundary conditions. Except for the DWR re-definition of the positive x direction, the η evolution, Figures 6.5a and 6.8a, and the Q evolution, Figures 6.6a and 6.9a, are indeed close.

A similar comparison with the CCW predictions, 6.3a and 6.3a, is less satisfactory. The differences are considerable, and apparently more than might be attributable to the different channel shapes[5], rectangular for CCW and trapezoidal for DWR and RMA. The source of this discrepency seems to ????

REFERENCES

Bode, L. and R. J. Sobey (1984). Initial transients in long wave computations. *Journal of Hydraulic Engineering 110*, 1371–1397.

[5]An ESTFLOW simulation with the CCW rectangular channel geometry resulted in $\eta(x,t)$ and $Q(x,t)$ evolutions very similar to the DWR and RMA predictions for a trapezoidal channel. The ESTFLOW predictions for the trapezoidal channel geometry were visually identical to the DWR and RMA predictions.