Chapter 5

H2: Transient Evolution of an Initial Mound

Contents

5.1 Problem Specification

H2 Transient evolution of an initial mound in an open-ended channel on a horizontal bed

Focus propagation of wave, open boundary conditions.

Channel bed is horizontal from downstream point F to upstream point L (see Figure 4.1). The trapezoidal channel bed width B is 10 ft. At F, $x_F = 0$ ft and $Z_F = +1.00$ ft. At L, $x_L = 10{,}000$ ft and $Z_L = +1.00$ ft. Channel friction factor is constant at Darcy-Weisbach $f = 0.03$ or Manning $n = 0.02$.

The initial conditions at $t = 0$ are

$$\eta(x,0) = 5 + 0.5\exp\left[-c\left(\frac{x-5000}{1500}\right)^2\right] \quad \text{ft} \tag{5.1.1}$$

$$Q(x,0) = 0 \tag{5.1.2}$$

where $c = \ln 2 = 0.6931$.

Use a fixed computational space step $\Delta x = 500$ ft and a fixed computational time step $\Delta t = 30$ s.

Compute and write to file in the STANDARD FORMAT the initial conditions at $t = 0$ and the model predictions for every time step to $t = 50\Delta t$.

5.2 Background

This problem is a variation on a text book solution of the classical wave equation. One simplified form of the hydrodynamic Equations 2.4.1 and 2.4.2 neglects both advective inertia and friction, and assigns both $b = b_0$ and $A = A_0$ as constant to linearize the residual equations. Eliminating Q from the two equations by cross-differentiation reduces them to the classical text book wave equation

$$\frac{\partial^2 \eta}{\partial t^2} = C_0^2 \frac{\partial^2 \eta}{\partial x^2} \tag{5.2.1}$$

where $C_0 = [gA_0/b_0]^{1/2}$ is the wave speed. Alternatively, eliminating η yields the same equation in Q.

The d'Alembert solution (Morse and Feshbach 1953) of these equations is

$$\begin{aligned} \eta(x,t) &= f(x - C_0 t) + g(x + C_0 t) \\ Q(x,t) &= F(x - C_0 t) + G(x + C_0 t) = b_0 C_0 \left[f(x - C_0 t) - g(x + C_0 t) \right] \end{aligned} \tag{5.2.2}$$

With initial conditions as Equation 5.1.1, $f(s) = g(s) = \frac{1}{2}\eta(x, 0)$.

This analytical solution is shown in Figure 5.1, to the same limits as the expected numerical solution. The response pattern is physically very explicit. The initial mound is split exactly in two, with each part retaining its exact shape, one part propagating to the right at speed C_0 and the other to the left at the same speed C_0. The $Q(x,t)$ response is very similar. After the passage of the disturbances, the solution field returns to quiescence.

Without the linearization and retaining the advective acceleration and friction terms, the response pattern remains much the same as Figure 5.1. The initial mound will be split exactly in two, one part propagating to the right and the other to the left. There will be some evolution of the mounds as they propagate, but the solution field will return to quiescence, after passage of the disturbances. An illustration of the expected response from the complete field Equations (2.4.1 and 2.4.2) is provided by predictions from the ESTFLOW[1] code. The ESTFLOW-predicted solution field evolution is shown in Figure 5.2. At these scales, the difference from the Figure 5.1 linearized analytical solution are not especially evident. The difference are much clearer in Figure 5.3, which shows just η profiles at selected times; the solid line is the Equation 5.2.2 analytical solution and the crosses are the ESTFLOW numerical solution prediction. At this scale, there are difference between the predicted responses from the complete field Equations (2.4.1 and 2.4.2) and

[1] This is a flexible instructional code developed and maintained by Professor Sobey. The numerical algorithm in the current version is a quadratic Method of Lines, together with an adaptive time step size Runge-Kutta code for time integration. It has a wide range of boundary condition options.

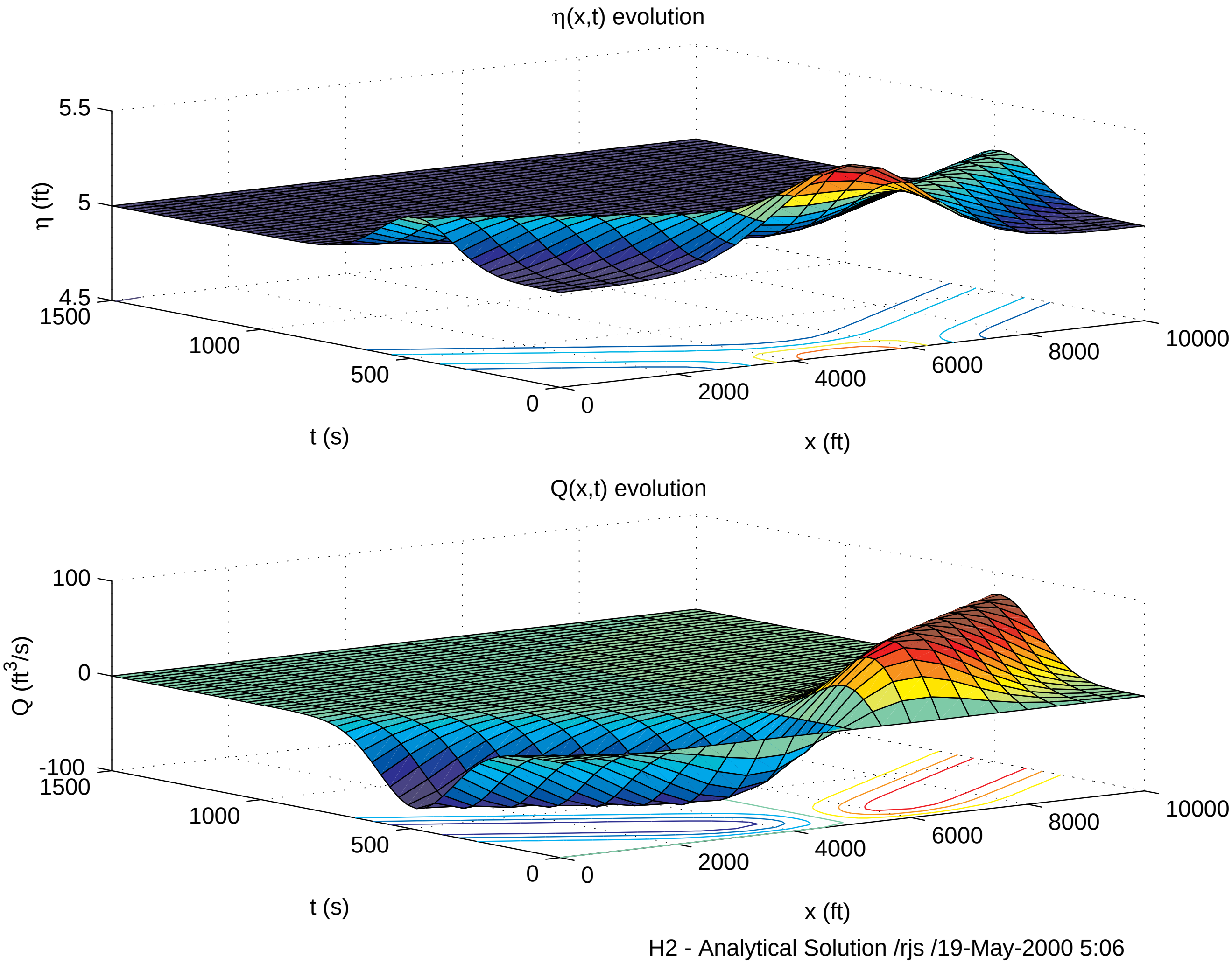

Figure 5.1: H2 linearized analytical solution field evolution

from the simplified classical wave equation (Equation 5.2.1). The most significant difference is the wave speed, the numerical solution to the complete field equations predicts that the propagation speed of the disturbance is marginally slowed. Overall however, the character of the response pattern remains unchanged as expected. The initial mound will be split exactly in two, one part propagating to the right and the other to the left. There will be some evolution of the mounds as they propagate, but the solution field will return to quiescence, after passage of the disturbances.

Despite the unique and very explicit view of WAVE PROPAGATION offered by this problem, the more significant focus is OPEN BOUNDARY CONDITIONS, and their representation in the code. There is no internal forcing in the hydrodynamic equations, and predicted response patterns are driven by the open boundary conditions (especially) and by the initial conditions. If the open boundary conditions are not correct, then the predicted solution can not be correct; it will be a solution to a potentially very different problem. In application, neither the numerical algorithm

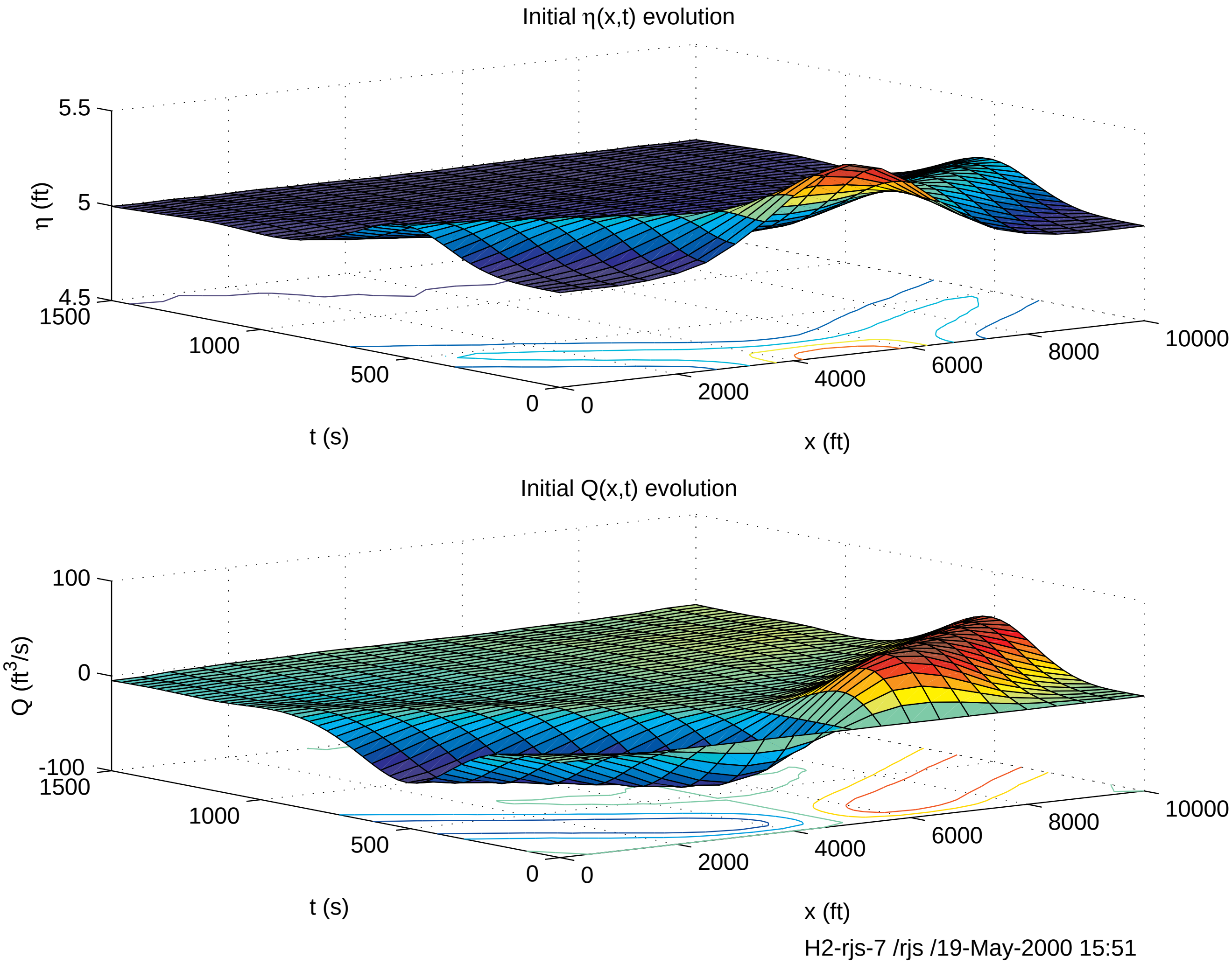

Figure 5.2: H2 ESTFLOW-predicted solution field evolution.

nor the coding are the crucial issue. The open boundary conditions are mostly THE important issue in whether a numerical prediction is an appropriate prediction of a specific physical problem.

The numerical solution restricts the extent of the solution domain in x. Physically, the left-propagating wave must pass completely through the left boundary. Similarly for the right-propagating wave at the right boundary. Any boundary condition that does not completely achieve this will reflect at least part of the wave back into the solution field. This will be seen as a variation from complete quiescence in the solution field after impact of the initial disturbance at the boundaries. Regular boundary condition impose η or Q, and will reflect part of the wave back into the solution field. The ESTFLOW solution in Figures 5.2 and 5.3 has radiation open boundary conditions (Bode and Sobey 1984); the disturbances pass through the open numerical boundaries at x_F and x_L without apparent reflection.

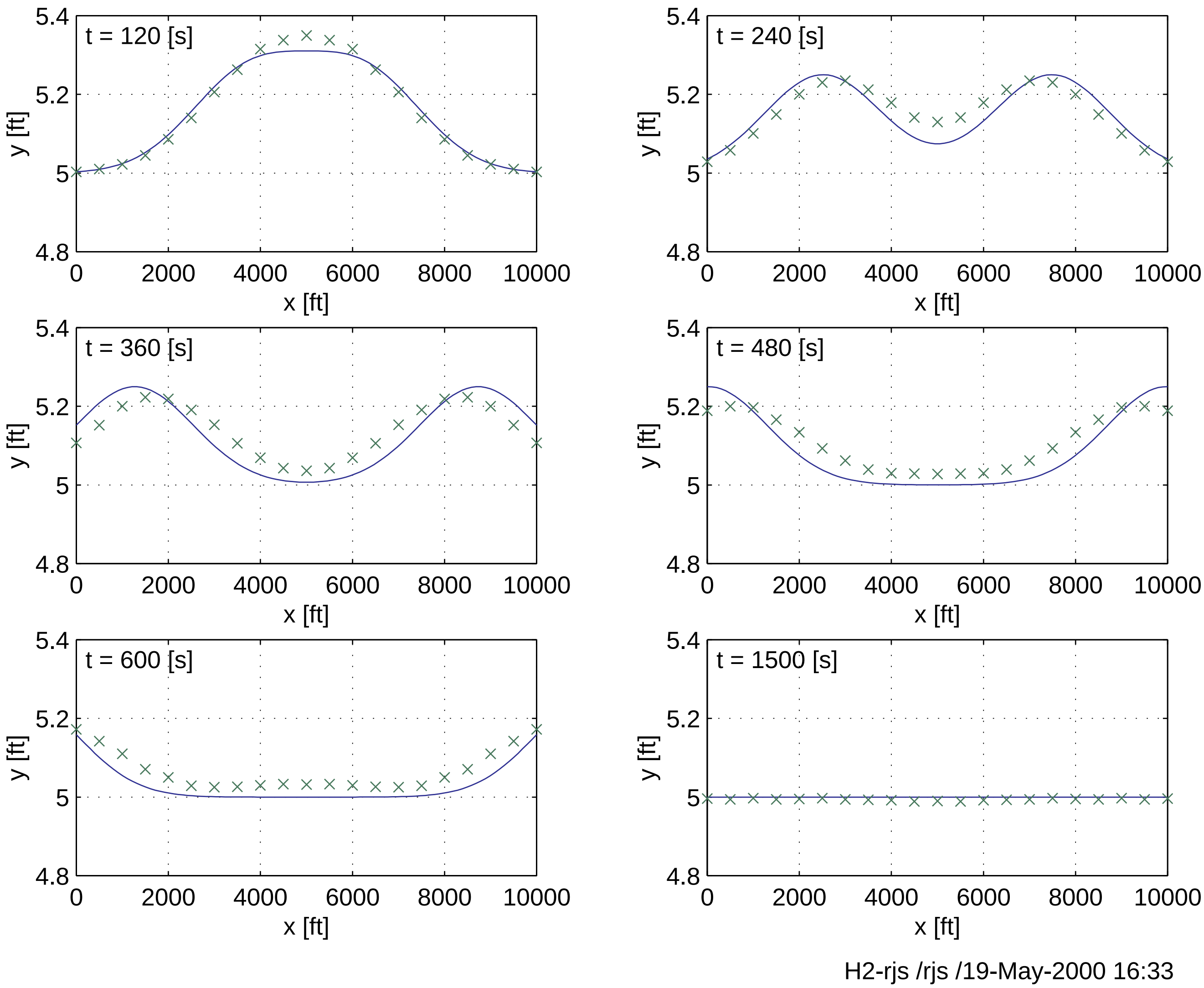

Figure 5.3: H2 ESTFLOW-predicted η solution evolution. Solid line is simplified analytical solution, crosses are numerical solution.

5.3 Contra Costa Water District

The CCW-predicted solution field evolution is shown in Figures 5.4 and 5.5. The expected response is clearly not achieved in Figure 5.4, but the details are much clearer in Figure 5.5. The immediate response at $t < 200$ s does follow the expected mound split and left and right propagating waves. But these waves are unable to pass through the numerical boundaries, and are reflected. The result is a sloshing bathtub-style response.

Apparently, the CCW model does not have radiation boundary conditions as an option[2]. The

[2]The following commentary was provided by CCW (Shum, 27 April 2001): "This problem requires that a radiation-type boundary condition be incorporated in the numerical code. However, none of the models tested were

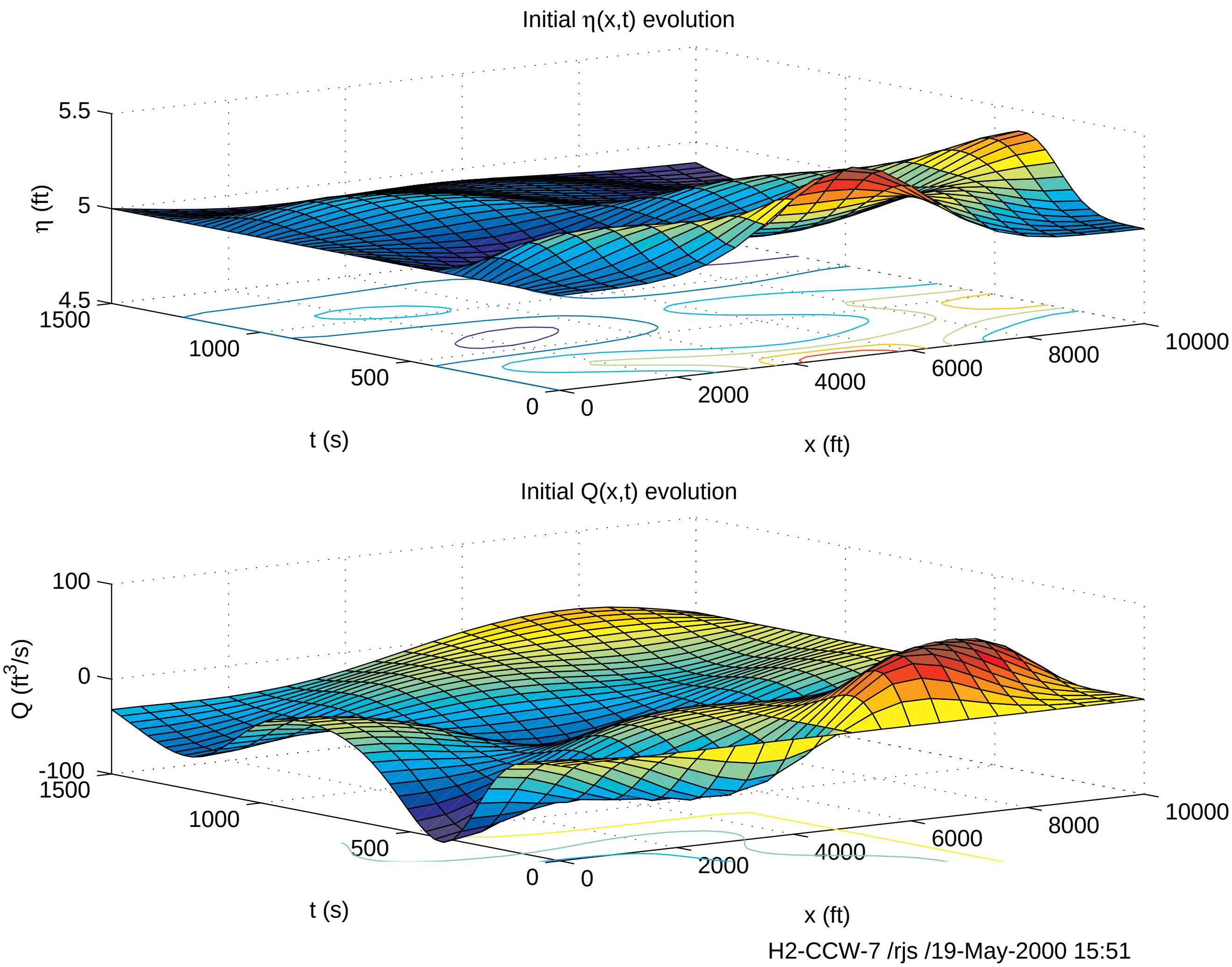

Figure 5.4: H2 CCW-predicted solution field evolution.

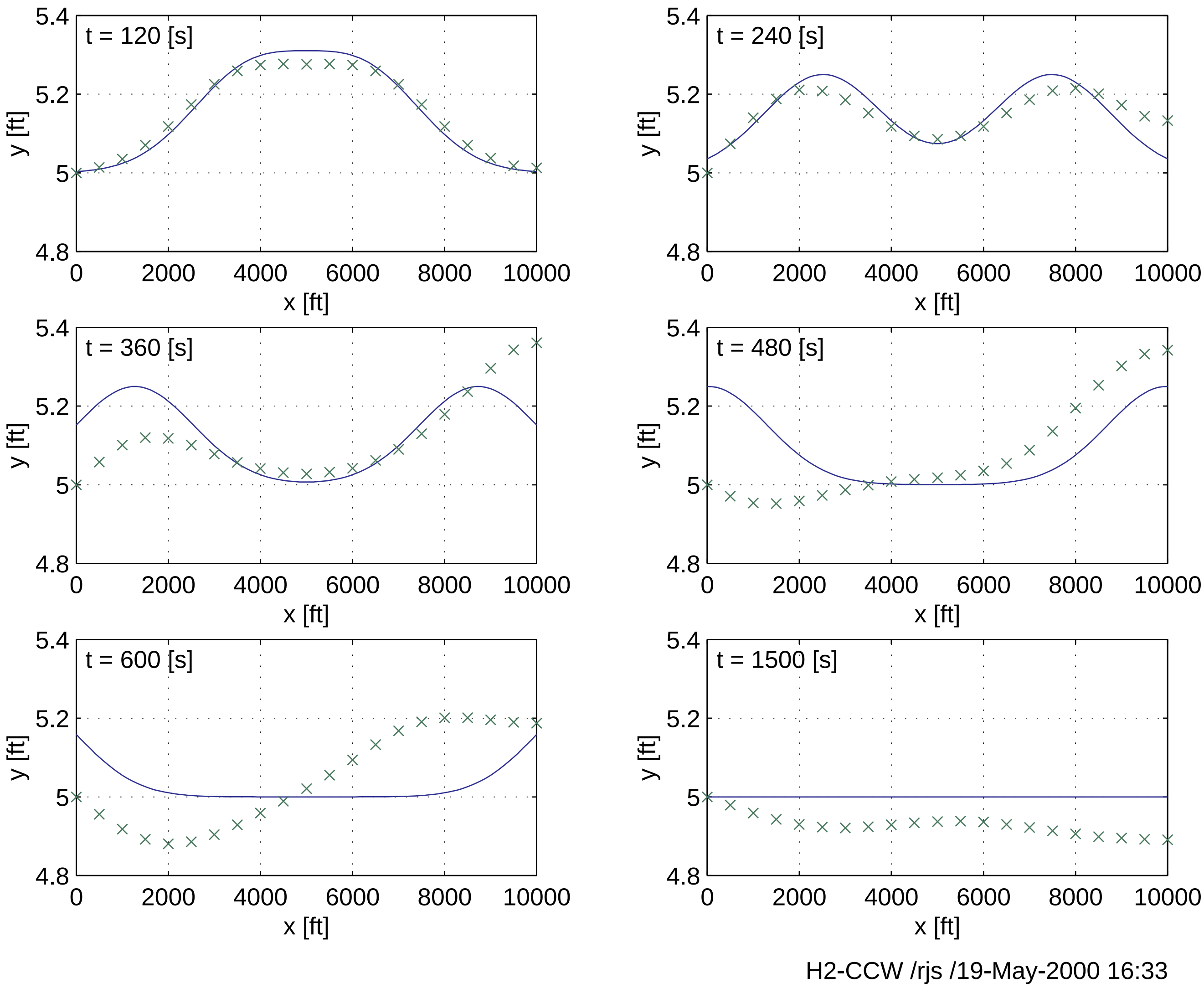

Figure 5.5: H2 CCW-predicted η solution evolution. Solid line is simplified analytical solution, crosses are numerical solution.

CCW choice of open boundary conditions appears to be

- $\eta(x_F, t) = 5$ ft and constant
- $Q(x_L, t) = 0$, i.e. no flow

These are not symmetric, so that the predicted sloshing response is consequently asymmetric. Given this choice of boundary conditions, the predicted response follows the expected pattern. Both the propagation properties and the response to open boundaries are appropriate.

Figure 5.6 shows the time history of the mass and momentum balances at location $x = 3{,}000$ ft. Both mass and momentum are conserved at the selected location.

equipped with radiation boundary conditions. The justification offered for the inclusion of this problem appears to be the test of "free-modes propagation" in the numerical schemes. However, in the absence of the proper boundary conditions in the numerical scheme, the solutions would not be able to assess model performance. An alternative to significant code modifications to simulate such free-modes would be to expand the modeled area."

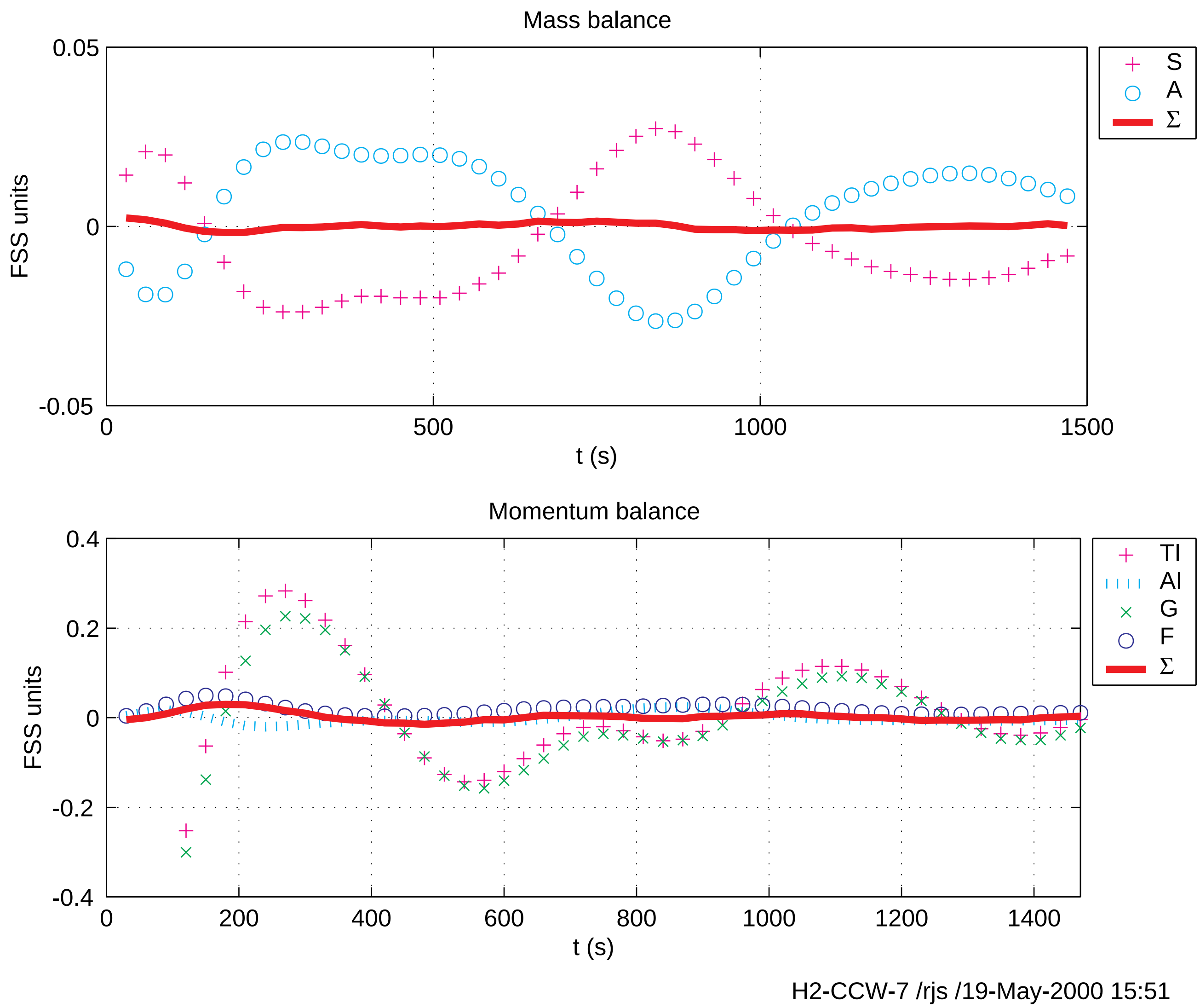

Figure 5.6: H2 CCW-predicted conservation balances at $x = 3{,}000$ ft.

5.4 Department of Water Resources

The DWR-predicted[3] solution field evolution is shown in Figures 5.7 and 5.8. The initial impression of these results is perfection. The mound separation and left and right propagating waves is exactly as expected. The propagation properties of this model are clearly appropriate.

It also appears that radiation style boundary conditions were imposed at x_F and x_L, and that these are very successful in passing the waves through the open boundaries without reflection. But in Figure 5.8 at $t = 1500$ s, the initial disturbance has not yet completely passed from the system. It seems that the DWR model does not have the option of radiation boundary conditions, and that this omission was accommodated by a redefinition the H2 problem that extended the solution domain sufficiently (50,000 ft) to the left and to the right so that the disturbances do not reach the boundaries in 1500 s, but reporting the results only in the range $x_F \leq 0 \leq x_L$. This redefinition clearly demonstrates the propagation objective, but avoids the open boundary condition objective of this problem.

Again, the propagation properties of this model are clearly appropriate, but the response to open boundaries is unproven[4].

Figure 5.9 shows the time history of the mass and momentum balances at location $x = 7{,}000$ ft. Both mass and momentum are conserved at the selected location.

[3]The DWR data file reports the computational time step Δt as 1 s; it was apparently not the specified 30 s, but 60 s. The time step has been changed to 60 s for the following analyses.

[4]The following commentary was provided by DWR (2 February 2001): "In this problem, a reflection-free boundary condition is specified. None of the models tested have the capability to represent such a boundary condition. This left the door open for participants to use their best judgment in solving the problem. We believe that our approach is a valid and appropriate solution to the problem."

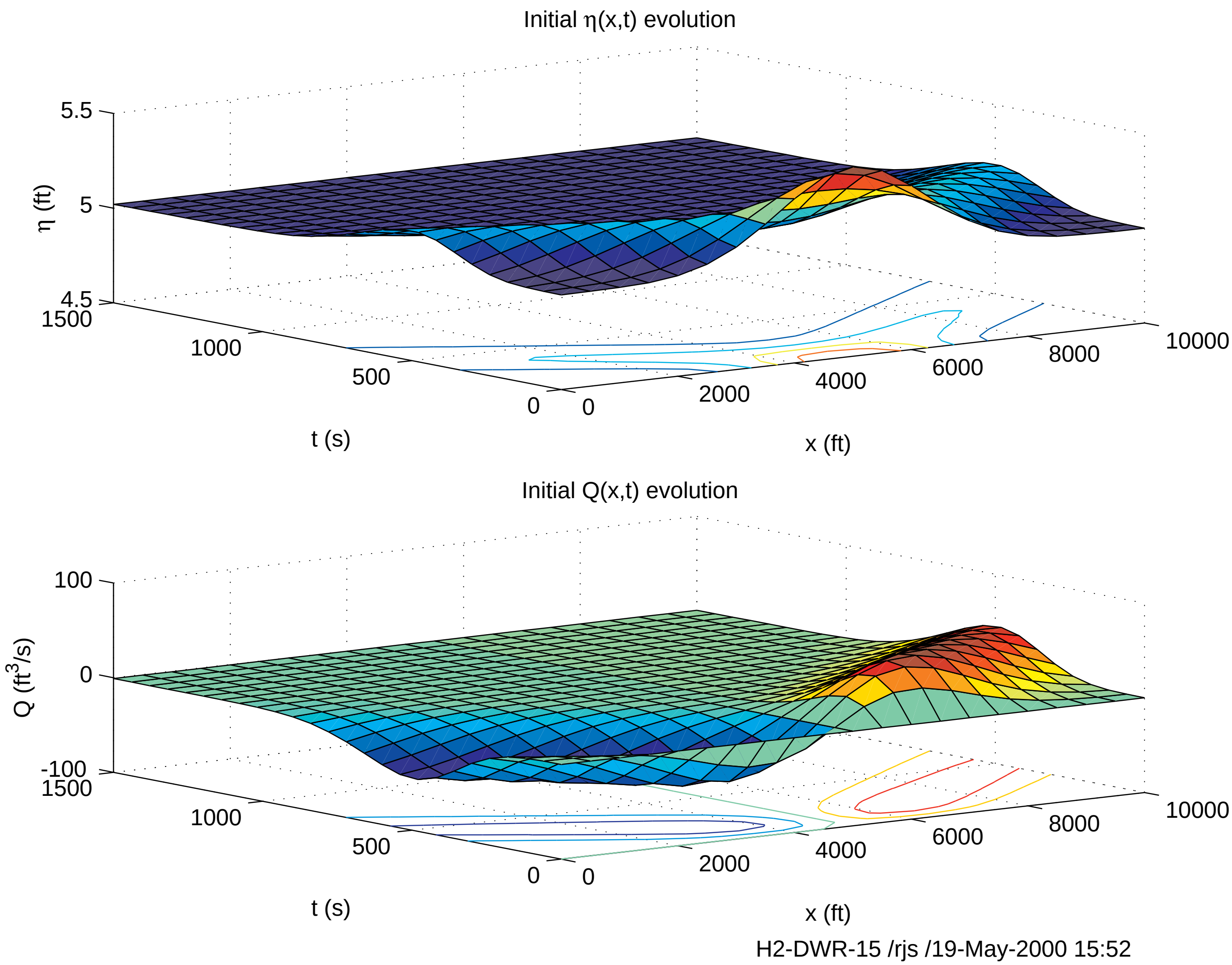

Figure 5.7: H2 DWR-predicted solution field evolution.

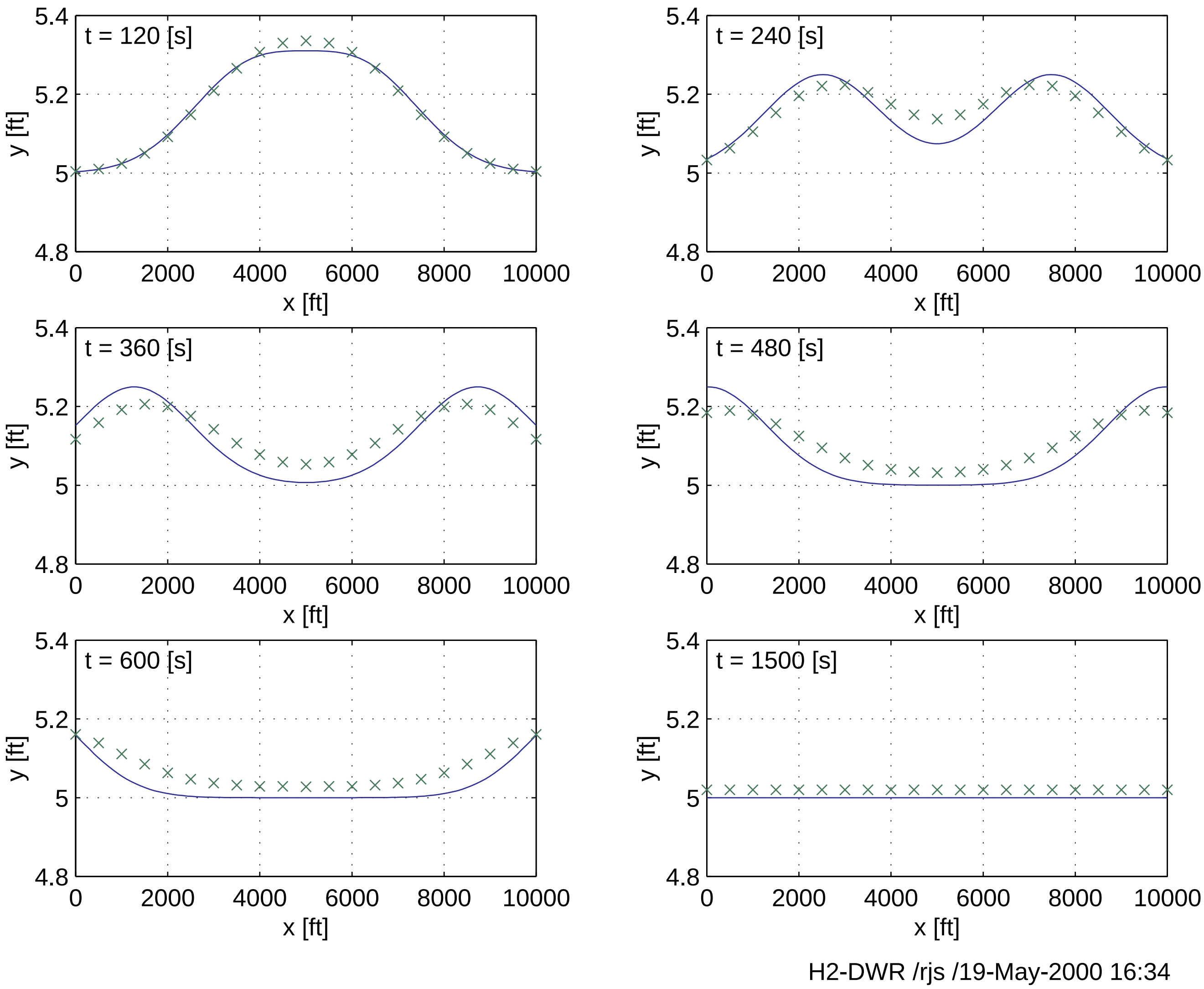

Figure 5.8: H2 DWR-predicted η solution evolution. Solid line is simplified analytical solution, crosses are numerical solution.

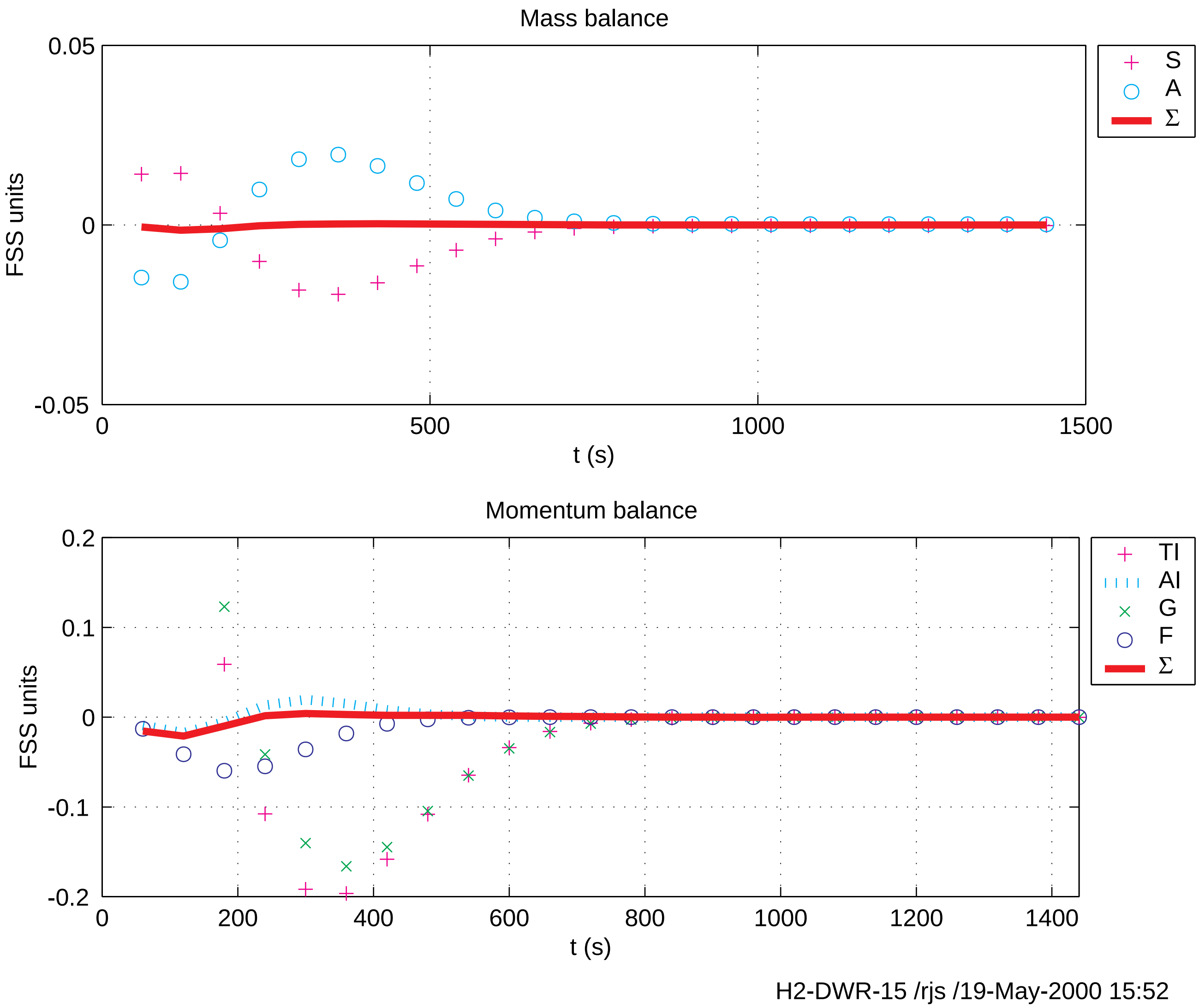

Figure 5.9: H2 DWR-predicted conservation balances at x = 7,000 ft.

5.5 Resource Management Associates

The RMA-predicted solution field evolution is shown in Figures 5.10 and 5.11. The expected response is clearly not achieved in Figure 5.10, but the details are much clearer in Figure 5.11. The immediate response at $t < 200$ s does follow the expected mound split and left and right propagating waves. Thereafter, symmetry is maintained but the impact of the boundary conditions become progressively apparent.

In addition, the left and right propagating waves are unable to pass through the numerical boundaries, and are reflected. The result is a sloshing bathtub-style response. Apparently, the RMA model does not have radiation boundary conditions as an option. The RMA choice of open boundary conditions appears to be

- $Q(x_F, t) = 0$ and
- $Q(x_L, t) = 0$,

i.e. no flow at either boundary. These are symmetric, so that the predicted sloshing response is also symmetric. This is seen very clearly in the water surface response in Figure 5.10a. Here, both the propagation properties and the response to open boundaries seem appropriate.

But there is an apparent problem with the flow response in Figure 5.10b. There are visual discontinuities distributed throughout the solution field. These are inconsistent with the smooth and simultaneously-predicted water surface response. These distributed discontinuities could be a coding error, a data reporting error, or an inconsistency in the formulation of the discrete equations. RMA argue[5] that it is the latter, and that the difficulty is controllable. This seems to be a viable explanation.

Figure 5.12 shows the time history of the mass and momentum balances at location $x = 3{,}000$ ft. There is a hint of a decaying oscillatory error in the mass balance. But momentum does not appear to be conserved. This is disappointing[6].

REFERENCES

Bode, L. and R. J. Sobey (1984). Initial transients in long wave computations. *Journal of Hydraulic Engineering 110*, 1371–1397.

Morse, P. M. and H. Feshbach (1953). *Methods of Theoretical Physics*. McGraw-Hill, New York.

[5] The following commentary was provided by RMA (DeGeorge, 19 January 2001): "There are oscillations observed in the flow solution, which we believe to be an undesirable numerical artifact of the mixed approximation FEM formulation referred to as "2 delta X waves". Some combinations of boundary conditions and grid resolutions can excite oscillations in the flow solution with a wavelength of two times the grid spacing (corner to mid-side), particularly with regularly spaced grids."

[6] The following commentary was provided by RMA (DeGeorge, 19 January 2001): "...the model is attempting to conserve mass and momentum in the integral sense, so the poor conservation at a single computation point reported by Dr. Sobey does not mean that there is an error in the formulation or code. It does suggest that the model is not giving a good answer to the problem. Refinement of the finite element mesh and/or time step will typically improve the results, reducing the magnitude of the oscillation to an insignificant level."

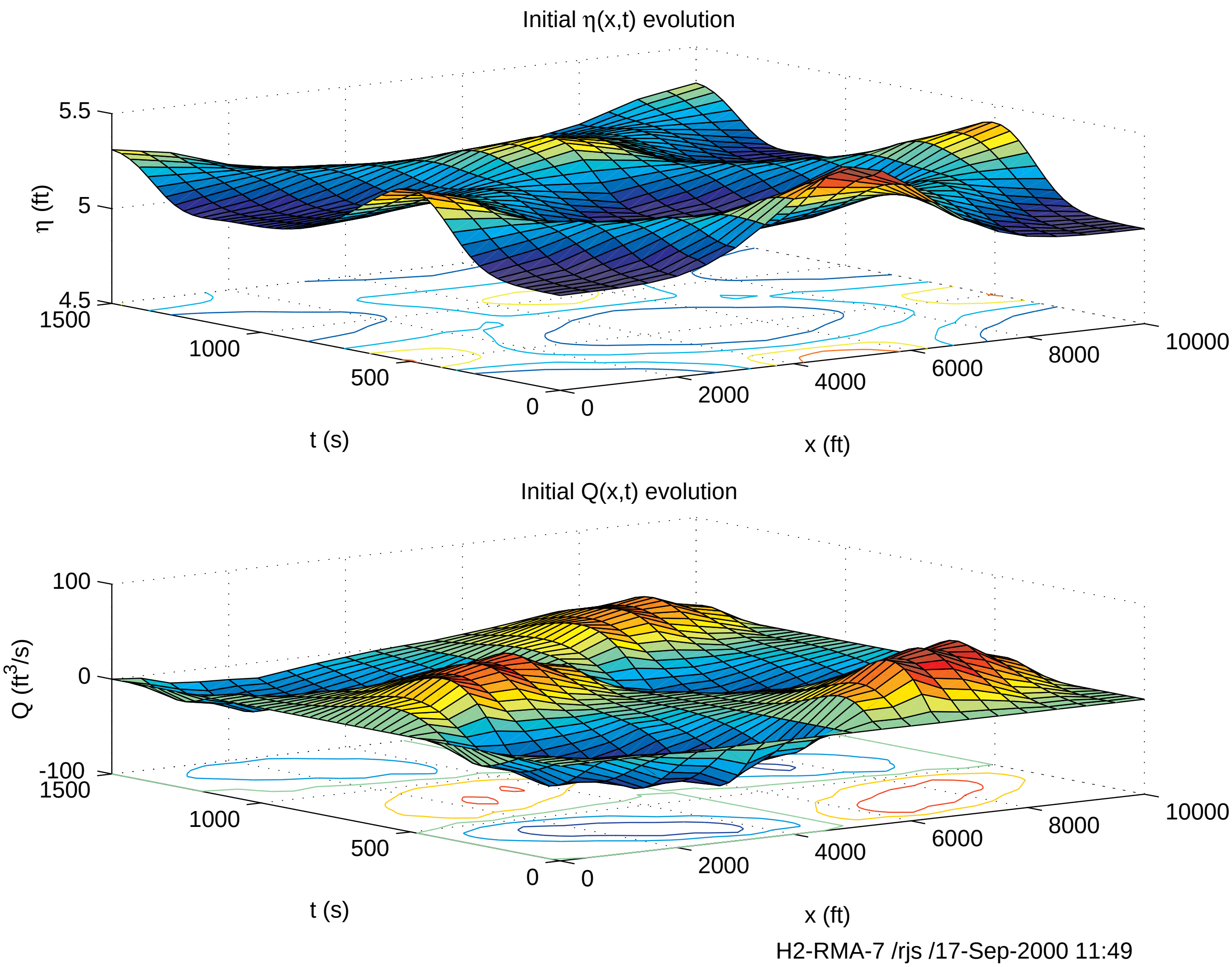

Figure 5.10: H2 RMA-predicted solution field evolution.

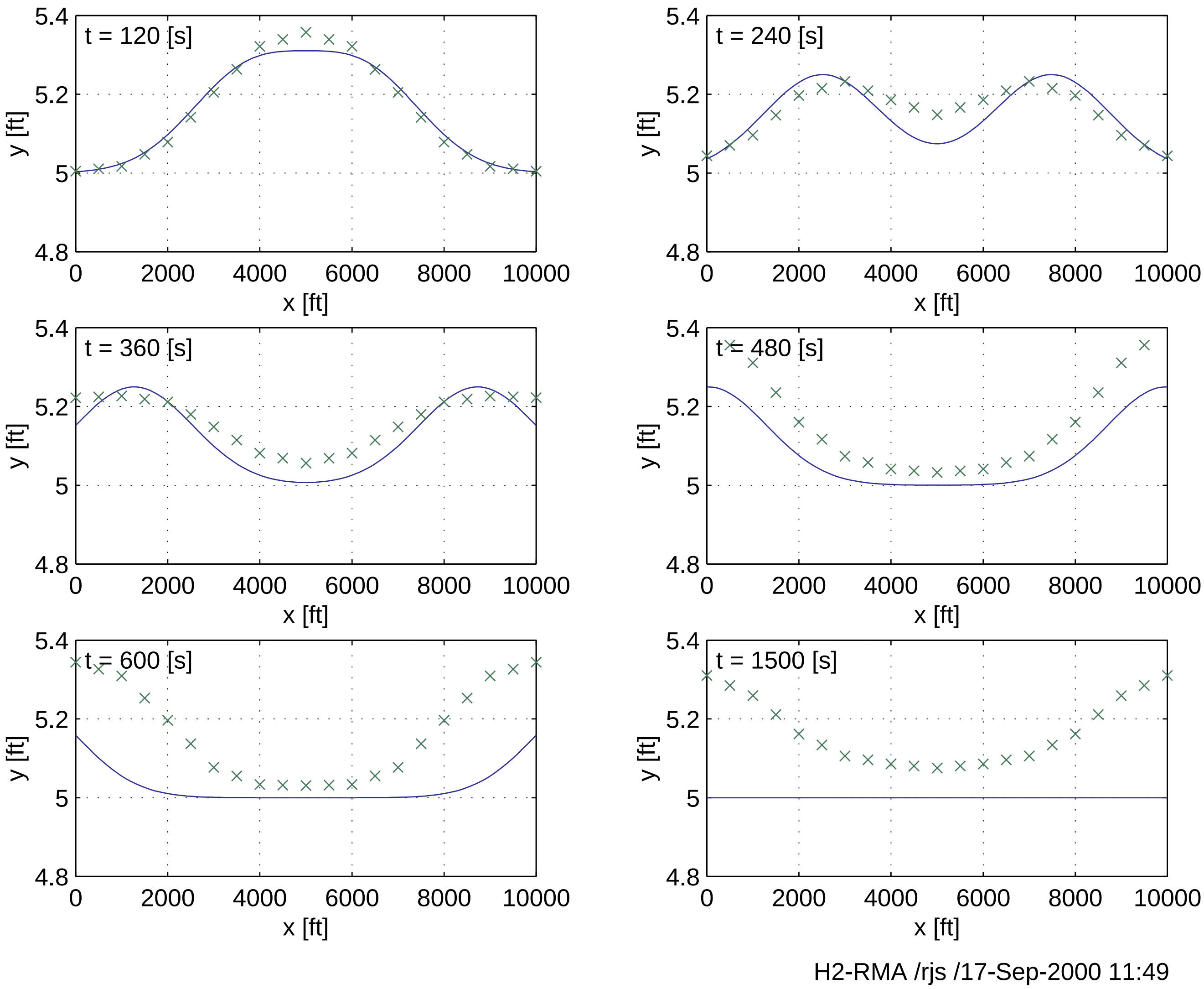

Figure 5.11: H2 RMA-predicted η solution evolution. Solid line is simplified analytical solution, crosses are numerical solution.

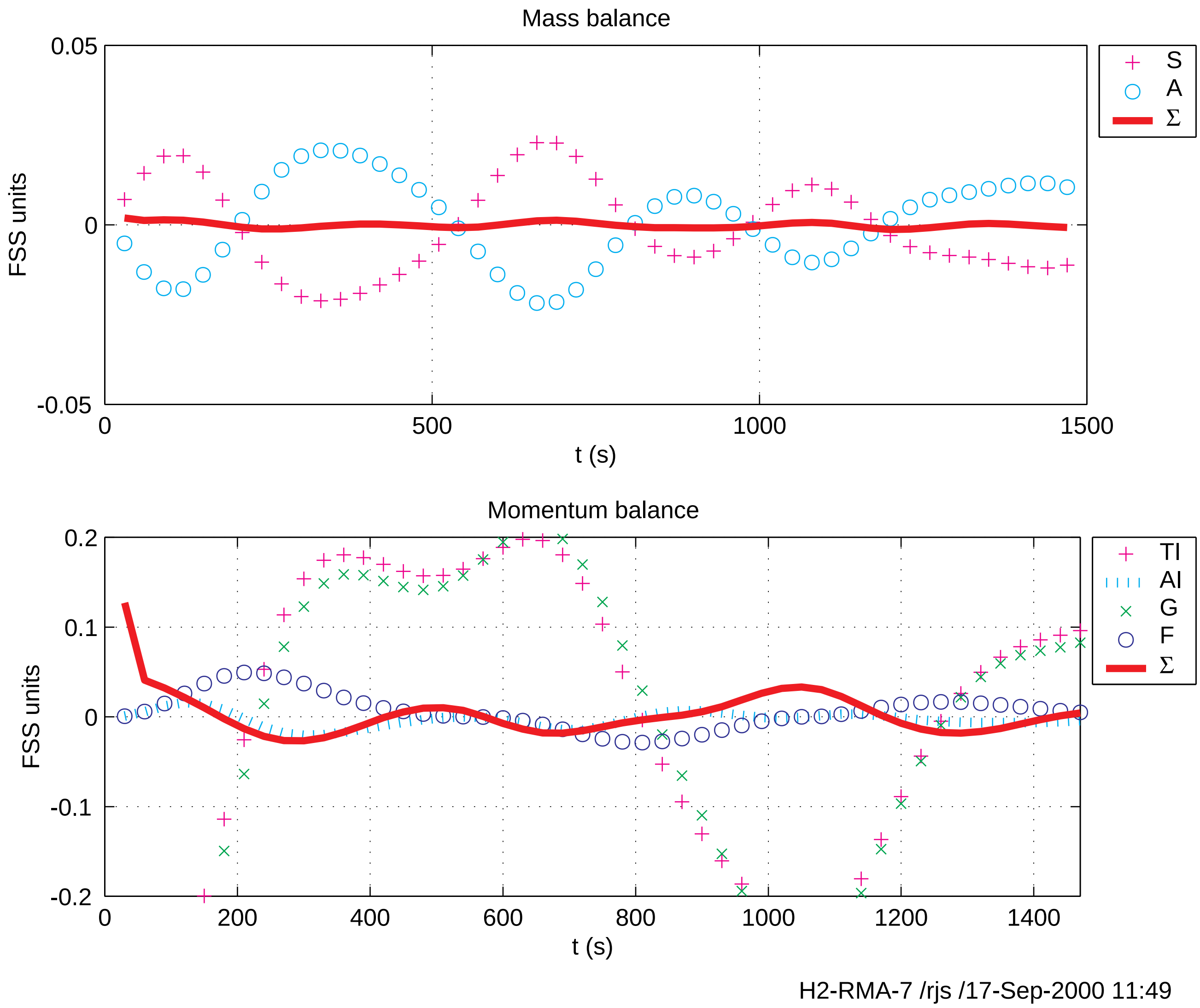

Figure 5.12: H2 RMA-predicted conservation balances at $x = 3{,}000$ ft.