Chapter 4

H1: Steady, uniform flow in an open-ended channel

Contents

4.1 Problem Specification

H1 Steady, uniform flow in an open-ended channel

Focus dominant momentum balance, numerical precision

Channel bed slopes linearly upwards from downstream point F to upstream point L (see Figure 4.1). The x axis is directed upstream from F to L. The trapezoidal channel bed width B is 10 ft. At F, $x_F = 0$ ft, $Z_F = +1.00$ ft and $Q_F = -200$ ft^3/s. At L, $x_L = 10,000$ ft, $Z_L = +3.00$ ft and $Q_L = -200$ ft^3/s. Channel friction factor is constant at Darcy-Weisbach $f = 0.03$ or Manning $n = 0.02$.

Use a fixed computational space step $\Delta x = 500$ ft and a fixed computational time step $\Delta t = 30$ s.

Choose near uniform flow initial conditions. Compute and write to file in the STANDARD FORMAT[1] the initial conditions at $t = 0$ and the model predictions for every time step to $t = 20\Delta t$.

[1]See Appendix C "Call for Phase A Model Evaluation"

Friction factor	$f = 0.03$	$n = 0.02$
h_n ft	4.64	4.73

Table 4.1: Normal Depth for H1 conditions

If necessary, continue this computation for sufficient additional time steps as is necessary to reach the STEADY STATE. Append this solution to the end of the output file.

Figure 4.1: Definition sketch for Problem H1.

4.2 Background

For steady flow in a uniform channel, the unsteady hydrodynamic Equations 2.4.1 and 2.4.2 reduce to

$$\frac{\partial Q}{\partial x} = 0 \tag{4.2.1}$$

$$0 = -gA\frac{\partial \eta}{\partial x} - \frac{\tau_0}{\rho}P \tag{4.2.2}$$

Accordingly, Q is constant, $\eta = \eta(x)$, a function of x only, and $\partial\eta/\partial x$ is constant and identical in magnitude to the bed slope. The uniform depth of flow under these conditions is called the normal depth h_n.

The normal depth can be computed directly from Equation 4.2.1, given a friction formula, Equation 2.4.3, the friction factor and the channel geometry. The normal depths are listed in Table 4.1.

This steady state balance should be the asymptotic solution for the channel. The usual momentum balance involves significant contributions from inertia, gravity and friction. In the steady state, the inertial term is theoretically zero, and the balance is between gravity and friction. But the potentially unsteady inertial term remains active.

Any error, regardless of the source, will likely drive an unsteady response, preventing the model from converging to the steady state. Potential errors may be

- math formulation - the field equations may not be identical to Equations 2.4.1 and 2.4.2.

- numerical algorithm - an inappropriate[2] discrete representation of the Equations 2.4.1 and 2.4.2. Difficulties with the nonlinear friction Equations 2.4.3 are possible.
- coding error - always possible in hundreds of lines of code.

The problem, as defined, is a compromise. With flow boundary conditions, the total mass of water within the reach is defined by the initial conditions; mass cannot escape from the flow domain, it can only be re-distributed. It would have been preferable to specify one water surface elevation and one flow boundary condition, which would permit flow to escape from the reach. But there was not a unique water surface elevation to ensure uniform flow, as the normal depth h_n depends on the channel friction model (see Table 4.1) and on the adopted channel geometry (see §4.3). This potential ambiguity did not cause a problem in the participant responses.

4.3 Contra Costa Water District

The CCW-predicted[3] solution field evolution is shown in Figure 4.2. The initial conditions at $t = 0$ are close but not identical to uniform flow. Initial transients at system natural frequencies (Bode and Sobey 1984) are generated; these decay naturally with friction. This is the expected response.

With time, the system is seen to approach a steady state profile with a constant water surface slope $\partial\eta/\partial x$ and a constant Q. Again, this is the expected response.

Steady state has been approached but not been reached[4] at $t = 600$ s. An approximation to the normal depth is the nodal-averaged water depth at this time. This is 3.96 ft. This does not correspond with the Manning value in Table 4.1 of 4.73 ft. But the CCW model is constrained to channel cross-sections that are rectangular; CCW set the rectangular width to $B = b = 23.24$ ft, for which h_n computes to 3.97 ft. Once again, this is the expected response.

Figure 4.3 shows the time history of the mass and momentum balances[5] at location $x = 3{,}000$ ft. The mass balance (the upper figure) shows each term in Equation 2.4.1; in the legend, S is storage, A is advection and $\Sigma = S + A$ is the mass balance, which should be identically zero. The model does not in fact conserve mass for $t < 200$ s. At these times, the solution evolution is dominated by the rapid initial transients that are poorly resolved by the adopted space scale ($\Delta x = 500$ ft) and time scale ($\Delta s = 30$ s). At later times, when the response is not resolution-challenged, mass is clearly conserved. This is the expected response.

The momentum balance (the lower figure) show each term in Equation 2.4.2; in the legend, TI is temporal inertia, AI is advective inertia, G is gravity and F is friction. Again, $\Sigma = TI + AI - G - F$ is the momentum balance, which should be identically zero. After the initial transients, momentum is clearly conserved. Also shown is the dominant gravity = friction ($G = F$) balance at uniform flow.

---

[2]The field equations are continuous partial differential equations. The numerical model generally solves discrete, linear and algebraic approximations.

[3]CCW re-defined the boundary condition at x_F from Q_F = -200 ft^3/s to $\eta_F = Z_F + h_n$. This is consistent with the spirit of the problem specification.

[4]The data files include a final output at $t = 90{,}000$ s, which is clearly steady state.

[5]Local conservation balances such as these have been adopted as a routine analysis tool. Details of the rationale and the analysis are given in Appendix D "Local Conservation Balances"

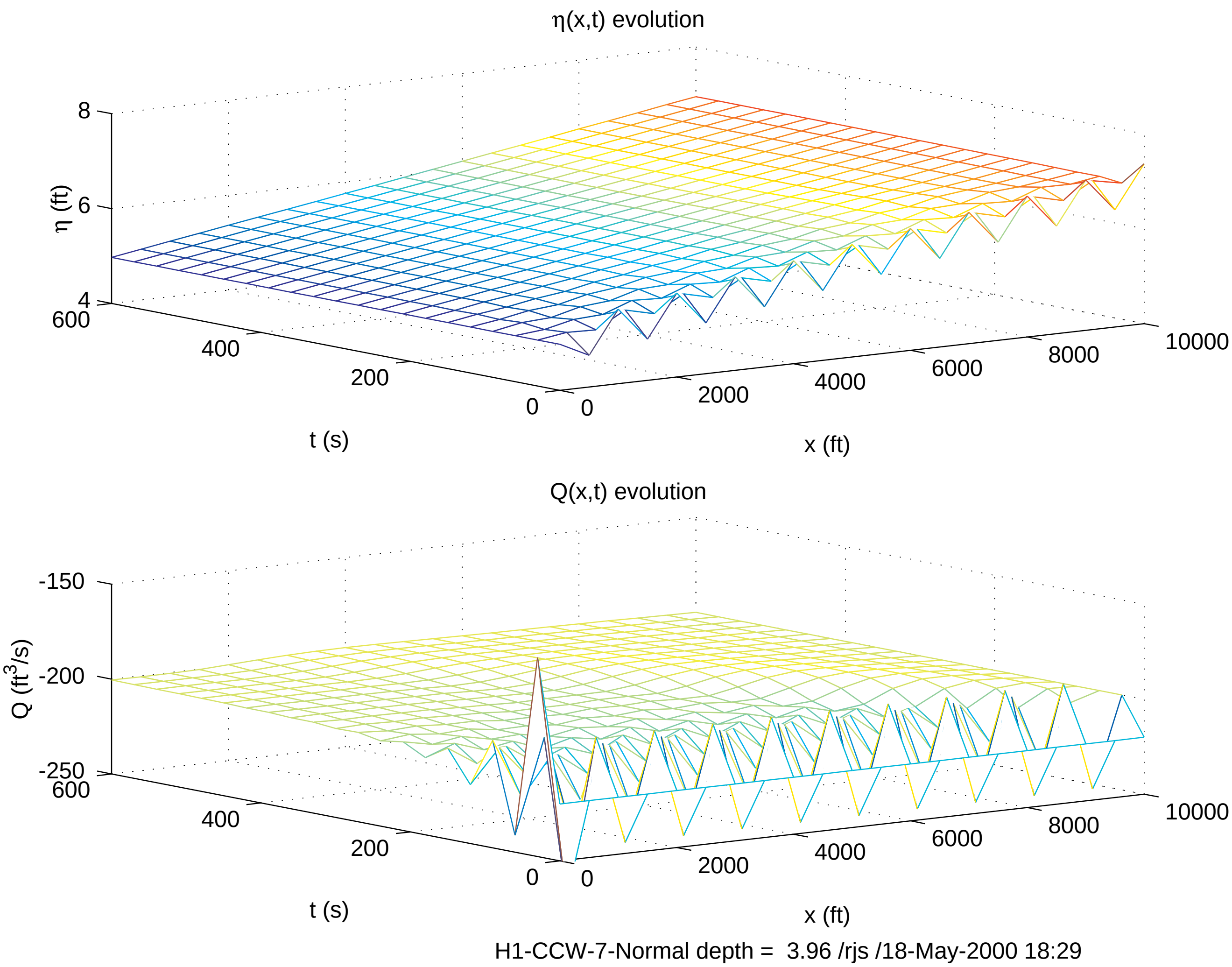

Figure 4.2: H1 CCW-predicted solution field evolution to 600 s.

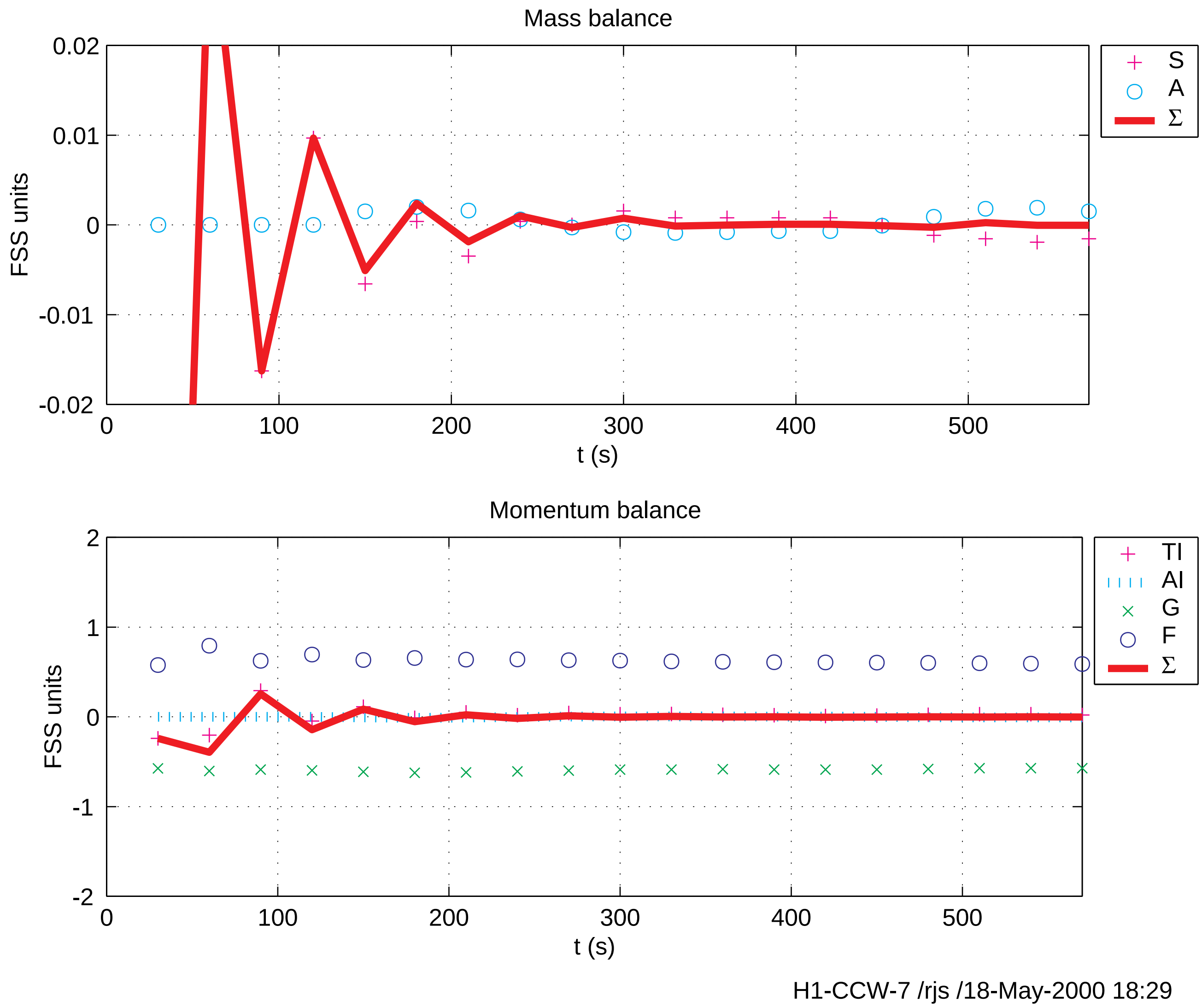

Figure 4.3: H1 CCW-predicted conservation balances at $x = 3{,}000$ ft.

4.4 Department of Water Resources

The DWR-predicted[6] solution field evolution is shown in Figure 4.4. The initial conditions at $t = 0$ for water level $\eta(x, 0)$ are close but not identical to uniform flow. The initial conditions for flow $Q(x, 0)$ are zero, so that there must be a very significant transition to uniform flow at a steady and uniform 200 ft^3/s. Such a large change is well within the predictive capabilities of the field equations, as is indeed demonstrated in this case. The steady state has clearly not been approached at $t = 600$ s.

Initial transients at system natural frequencies are generated (Bode and Sobey 1984); these decay naturally with friction. This is the expected response, and is well illustrated in Figure 4.5 where the predicted evolution has been extended to $t = 1500$ s. With time, the system is seen to approach a steady state profile with a constant water surface slope $\partial\eta/\partial x$ and a constant Q.

Steady state has been approached but not been reached at $t = 1500$ s. An approximation to the normal depth is the nodal-averaged water depth at this time. This is 4.74 ft, which corresponds almost exactly with the Manning value in Table 4.1; it does correspond exactly on further extension. Once again, this is the expected response.

Figure 4.6 shows the time history of the mass and momentum balances at location $x = 7{,}000$ ft. The mass balance (the upper figure) shows each term in Equation 2.4.1; in the legend, S is storage, A is advection and $\Sigma = S + A$ is the mass balance, which should be identically zero. The model does not in fact conserve mass in the region around $t = 400$ s. At these times, the solution evolution is dominated by the rapid initial transients that are poorly resolved by the adopted space scale ($\Delta x = 500$ ft) and time scale ($\Delta s = 60$ s). At later times, when the response is not resolution-challenged, mass is clearly conserved; see Figure 4.7. This is the expected response.

The momentum balance (the lower figure) shows each term in Equation 2.4.2; in the legend, TI is temporal inertia, AI is advective inertia, G is gravity and F is friction. Again, $\Sigma = TI + AI - G - F$ is the momentum balance, which should be identically zero. After the initial transients, momentum is clearly conserved. Also shown is the dominant gravity = friction ($G = F$) balance at uniform flow.

[6] A recurring difficulty with the DWR responses has been frequent re-definition of the test problems to accommodate apparent inflexibility in their code. The spatial coordinate x has been directed downstream and not upstream. Equally confusing has been a data file error in reporting the computational time step Δt. Δt is consistently reported in the data files as 1 s; it was apparently not the specified 30 s, but 60 s. Other changes vary with the problem.

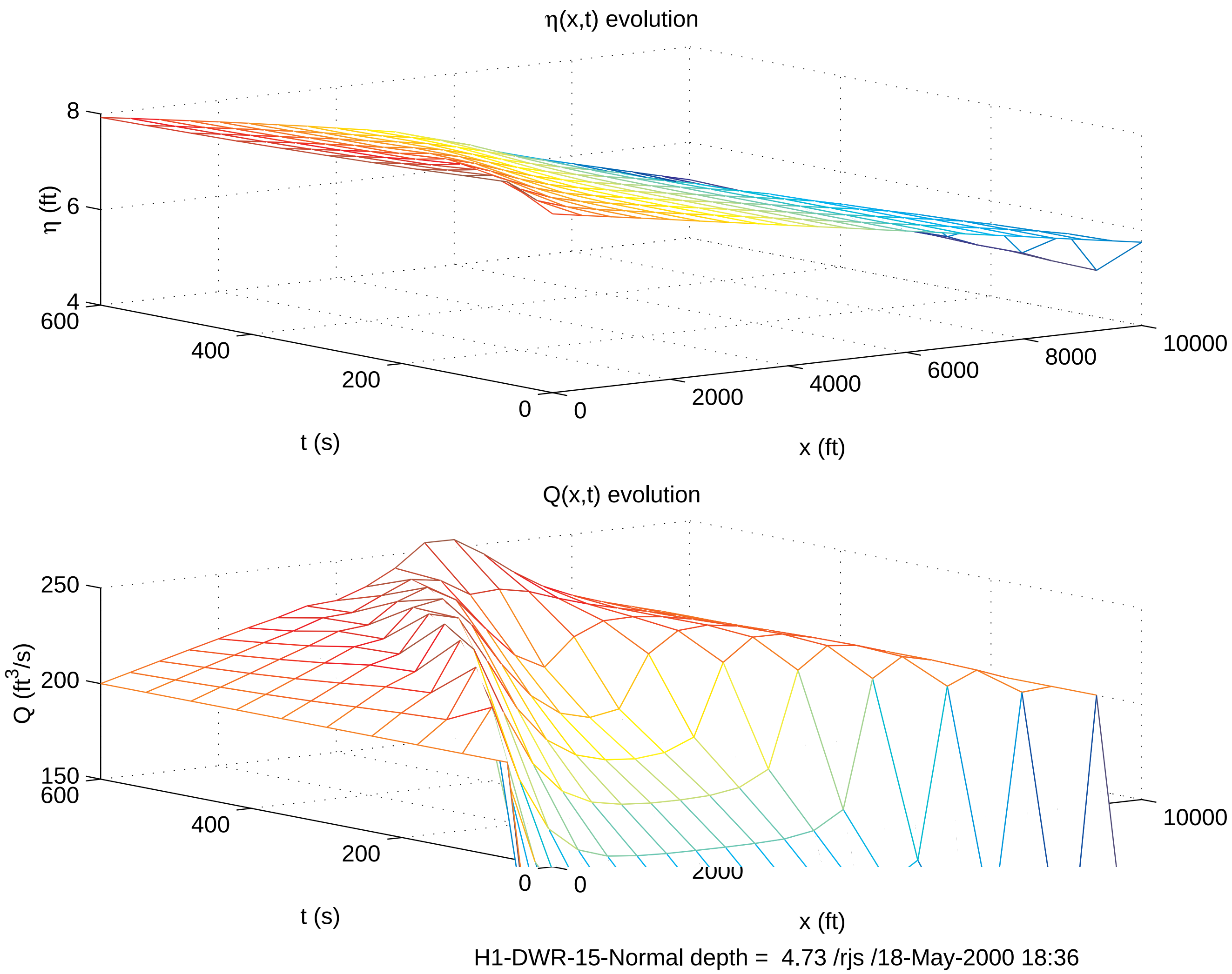

Figure 4.4: H1 DWR-predicted solution field evolution to 600 s.

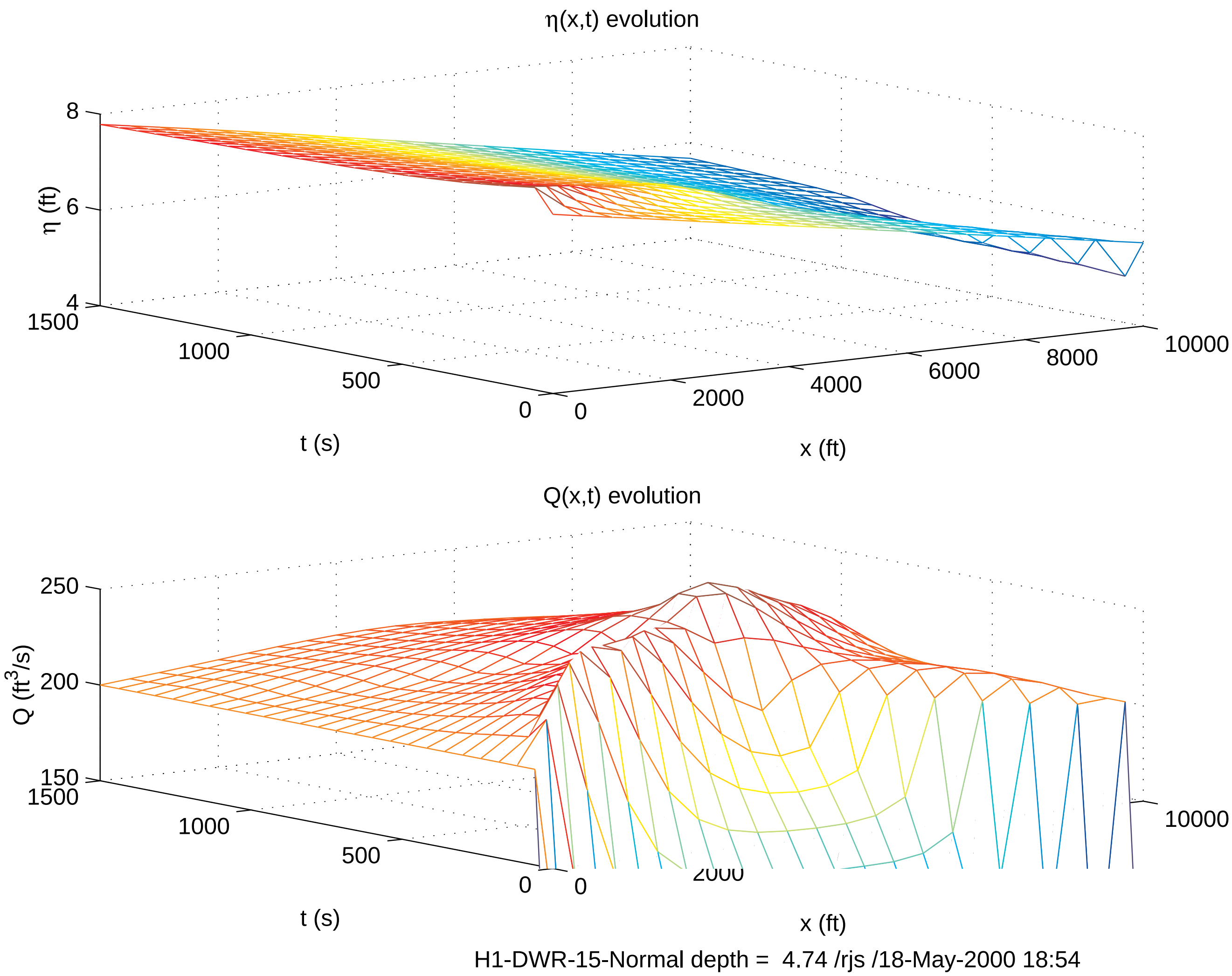

Figure 4.5: H1 DWR-predicted solution field evolution to 1500 s.

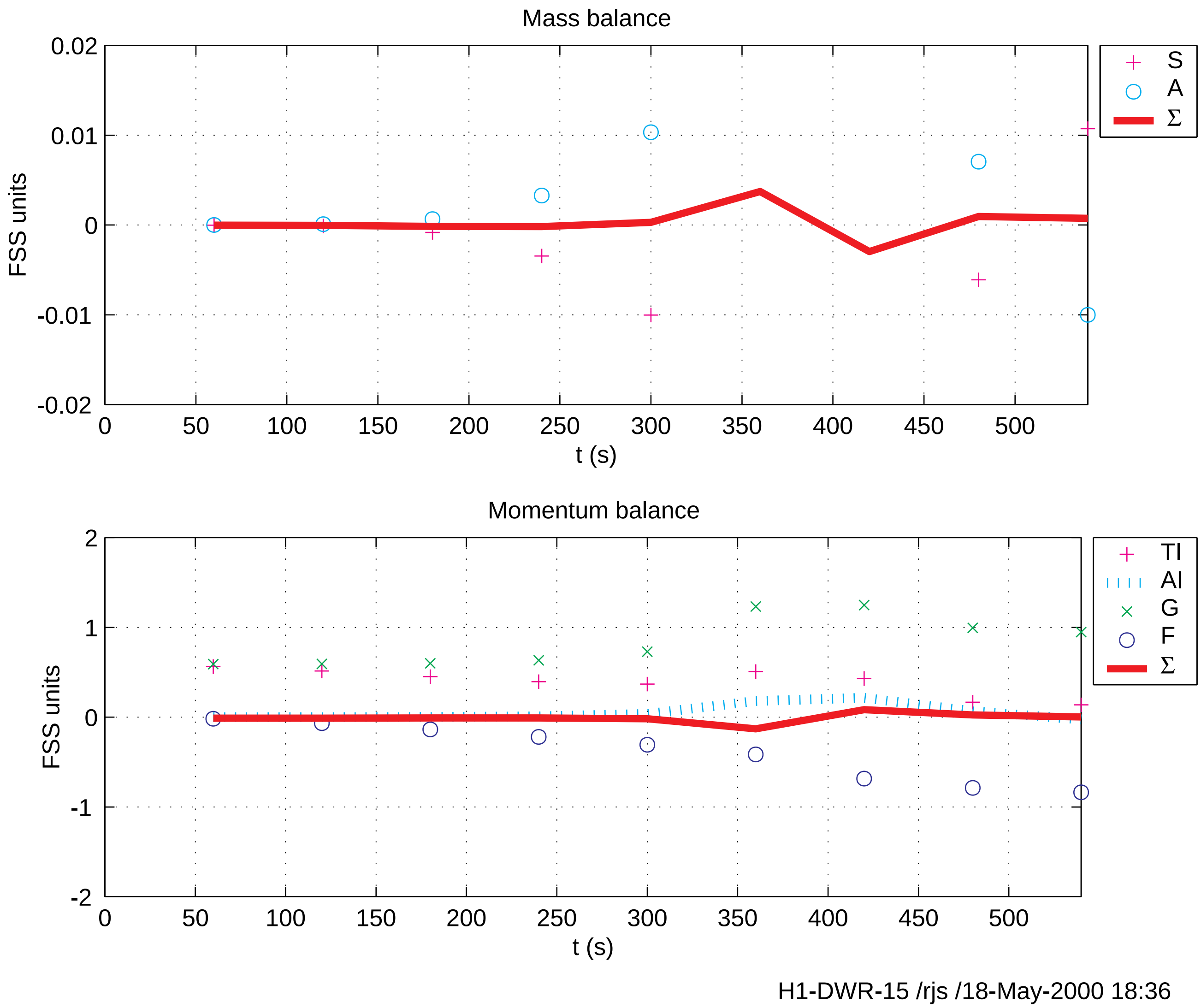

Figure 4.6: H1 DWR-predicted conservation balances to 600 s at x = 7,000 ft.

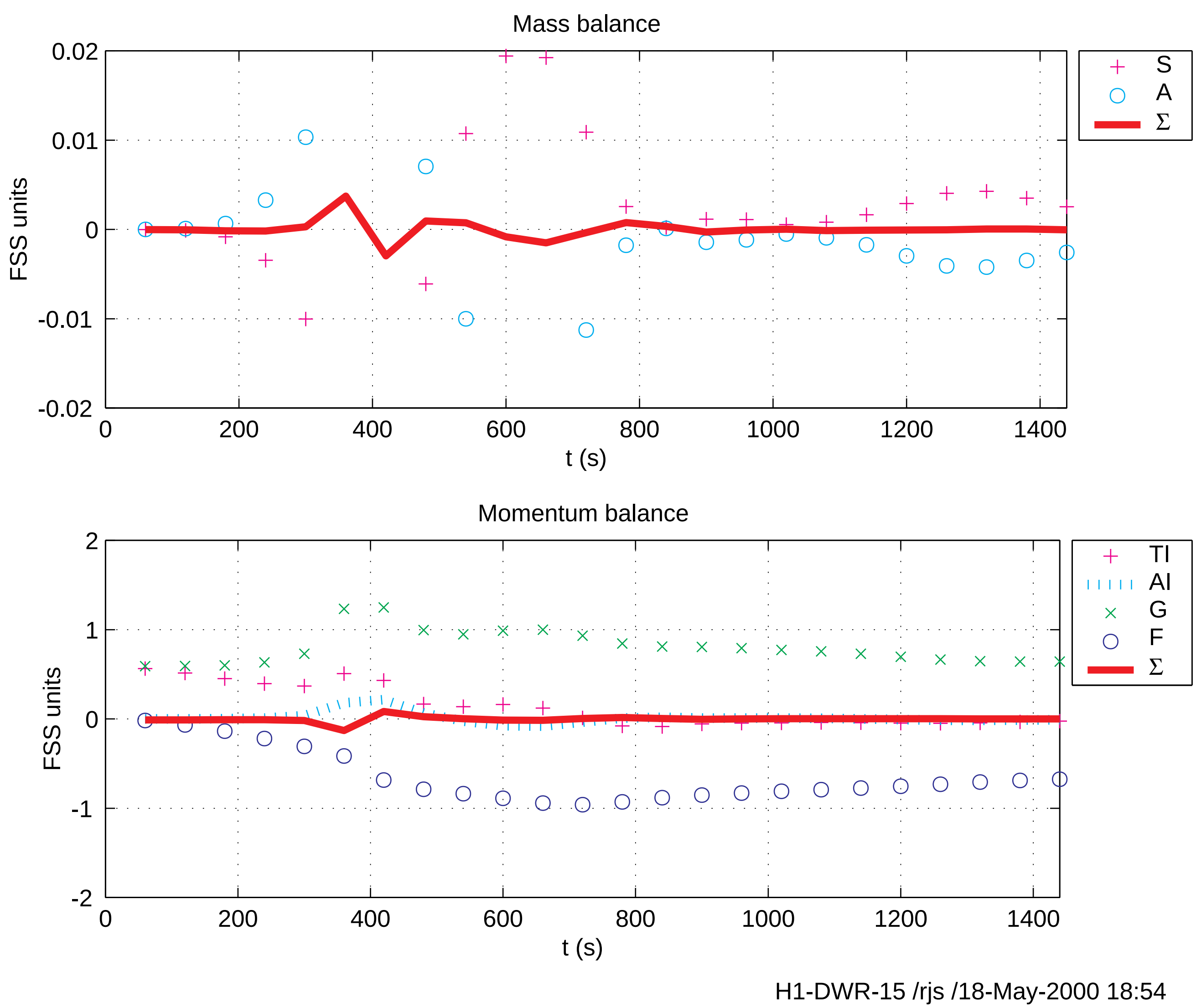

Figure 4.7: H1 DWR-predicted conservation balances to 1500 s at x = 7,000 ft.

4.5 Resource Management Associates

The RMA-predicted[7] solution field evolution is shown in Figure 4.8. The initial conditions at $t = 0$ are very close and almost identical to uniform flow. Some initial transient activity is apparent in the Q evolution; these decay naturally with friction. This is the expected response.

With time, the system is seen to approach a steady state profile with a constant water surface slope $\partial\eta/\partial x$ and a constant Q. Again, this is the expected response.

The steady state seems to have been approached but not completely reached at $t = 600$ s. An approximation to the normal depth is the nodal-averaged water depth at this time. This is 4.76 ft. This does not correspond exactly with the Manning value in Table 4.1 of 4.73 ft. Predictions to $t = 29{,}970$ s are provided. These are visually at uniform flow. Q is steady at -200.0 ft^3/s, but the steady depth (predicted by the "stage-elevation relation") at x_F is 4.7219 ft, whereas the steady depth (predicted by the code) at x_L is 4.7374 ft. This is not uniform flow, but a classical M_2 steady, gradually-varied flow profile.

While this is not uniform flow, it is quite close to uniform flow and there is no suggestion that this is not an expected and appropriate result. There seems to be a small inconsistency between uniform flow at 200 ft^3/s estimated from the code (at x_L) and from the "stage-elevation relation") at x_F.

Figure 4.9 shows the time history of the mass and momentum balances at location $x = 3{,}000$ ft. The mass balance (the upper figure) shows each term in Equation 2.4.1; in the legend, S is storage, A is advection and $\Sigma = S + A$ is the mass balance, which should be identically zero. The model does not in fact conserve mass for $t < 300$ s. At these times, the solution evolution is dominated by the rapid initial transients that are poorly resolved by the adopted space scale ($\Delta x = 500$ ft) and time scale ($\Delta s = 30$ s). At later times, when the response is not resolution-challenged, mass is clearly conserved. This is the expected response.

The momentum balance (the lower figure) shows each term in Equation 2.4.2; in the legend, TI is temporal inertia, AI is advective inertia, G is gravity and F is friction. Again, $\Sigma = TI + AI - G - F$ is the momentum balance, which should be identically zero. There is only a hint of the initial transients here, and momentum is clearly conserved. Also shown is the dominant gravity = friction ($G = F$) balance for the steady, gradually-varied (almost uniform) flow.

REFERENCES

Bode, L. and R. J. Sobey (1984). Initial transients in long wave computations. *Journal of Hydraulic Engineering 110*, 1371–1397.

[7] RMA re-defined the boundary condition at x_F from Q_F = -200 ft^3/s to $\eta_F(t) = Z_F + \tilde{h}_n(t)$, where $\tilde{h}_n(t)$ is defined by a "stage-elevation relation". In principle, this is consistent with the spirit of the problem specification.

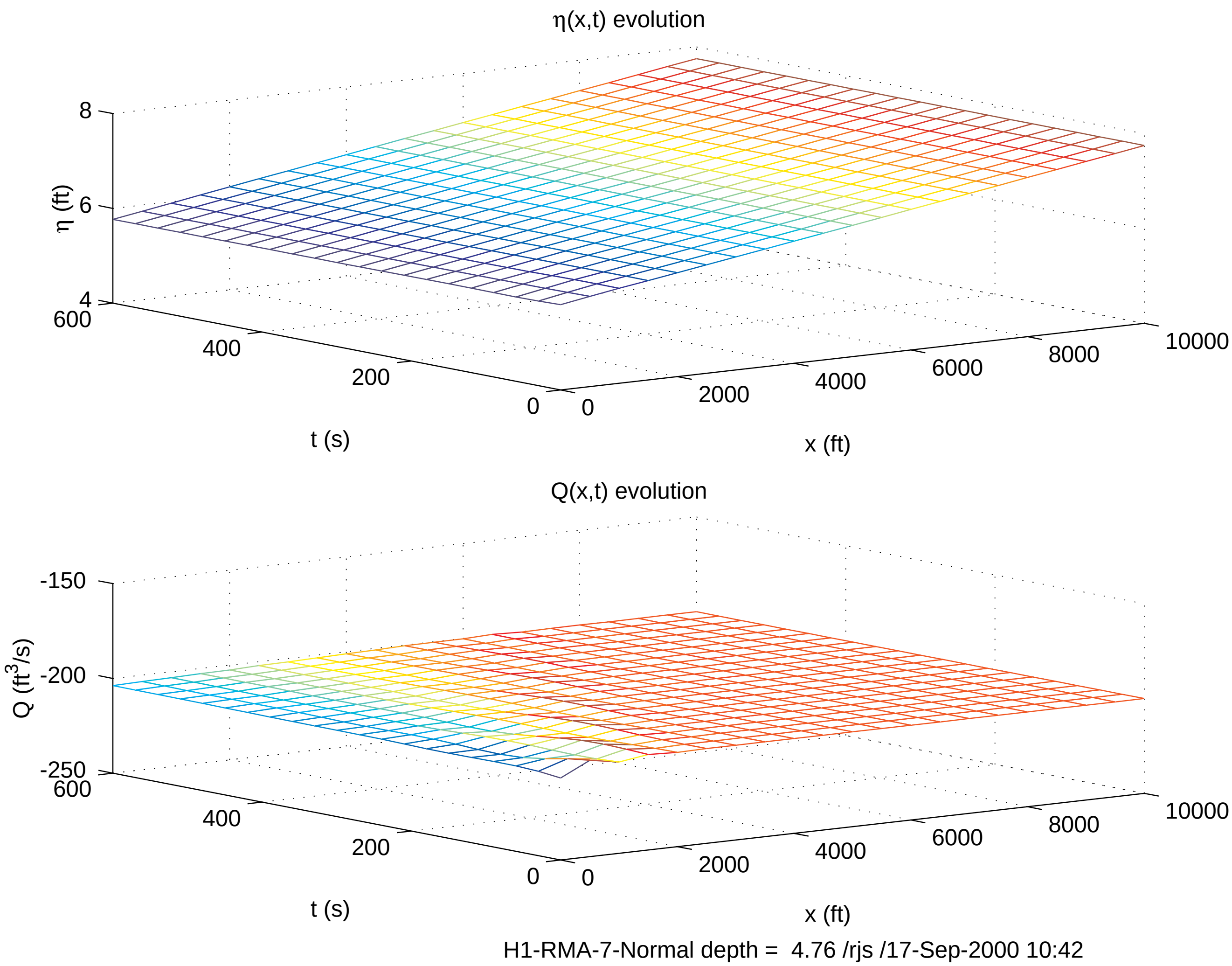

Figure 4.8: H1 RMA-predicted solution field evolution to 600 s.

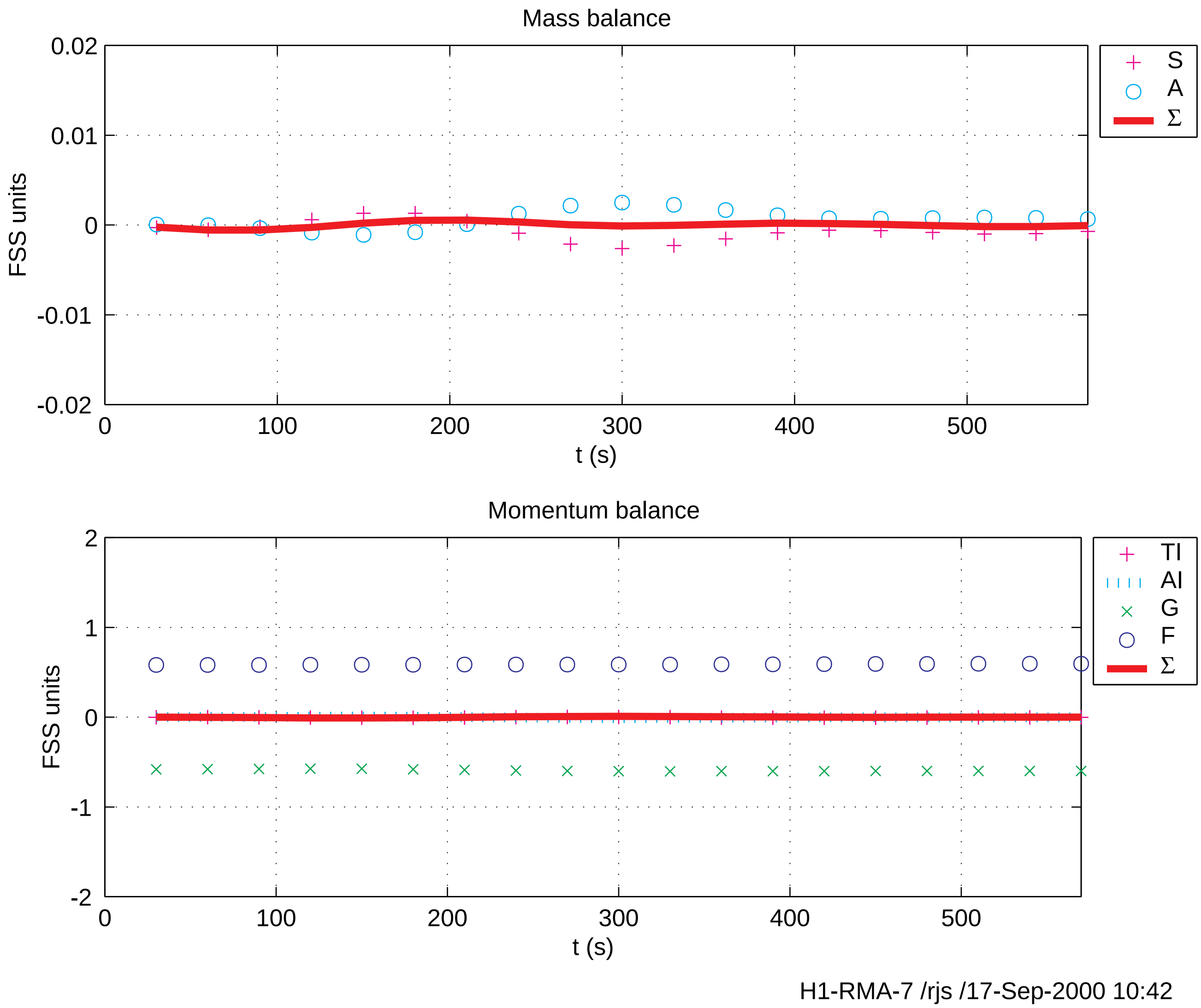

Figure 4.9: H1 RMA-predicted conservation balances at $x = 3{,}000$ ft.