INVESTIGATION OF METHODS USED IN THE INTEGRATED GROUNDWATER AND SURFACE-WATER MODEL

\[ Q_t = \begin{cases} C_R \left( h_t - h_z - \Delta t \right), & h_t - \Delta t > z_R \\ C_R \left( h_t - z_R \right), & h_t - \Delta t < z_R \end{cases} \]

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INVESTIGATION OF METHODS USED IN THE INTEGRATED GROUNDWATER AND SURFACE-WATER MODEL

by

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1. EXECUTIVE SUMMARY

This report summarizes findings of a review of the Integrated Groundwater and Surface-Water Model (IGSM) [Montgomery Watson, 1993] conducted at the University of California, Davis (UCD) Hydrologic Sciences Graduate Group at the request of the California Water and Environmental Modeling Forum (CWEMF). This report includes a review and analysis of the theory of IGSM and the IGSM code, several example problems, and conclusions and recommendations. This review found IGSM to be unreliable for a number of simple example problems. These example problems by no means test all aspects of the code, but are adequate for assessing some of the foundational methods of IGSM. Results identify and elucidate issues that would need to be solved prior to conducting a more extensive verification effort. This assessment by no means tests all aspects of IGSM; nor does it claim to identify all problems with the code. Based on the code structure and theoretical underpinnings that we have examined thus far, we anticipate identification of additional theoretical and implementation issues if further testing and verification are performed.

The CWEMF solicited comments on this report from a large group of IGSM users. Summaries of comments received and complete responses to those comments are provided in Appendix C. The original text of the comments, the test problem data sets and solutions, and source code for all simulations considered herein are provided on the CD that accompanies this document.

1.1 Principle Findings of this Review

Our analysis identified the following issues with the implementation and theoretical foundations of the IGSM solution methodology, any of which may lead to potentially significant errors in model solutions:

1. Improperly implemented head-dependent boundaries (e.g., general head, stream-aquifer, and drain boundary conditions).
2. Lack of a methodology to simultaneously converge coupled models, for example groundwater and surface-water models.
3. Explicit (non-standard) formulation of boundary conditions and head-dependent transmissivity.
4. Fixed monthly time step imposes a critical node spacing that severely limits applicability of the code and introduces potentially significant error into model solutions.

5. Incorrectly reported water budgets.

6. Lack of a method to ensure IGSM convergence to the governing nonlinear boundary value problem.

7. Undocumented additions to the code, lacking rigorous theoretical basis.

IGSM also lacks adequate documentation of the computer code and verification problems that demonstrate a working model.

1.2 Example Problems Results

Example problems described herein are simple and simulate common hydrologic phenomena. Problems were chosen based upon a review of the code to demonstrate some of the known issues that would need to be resolved prior to conducting a more comprehensive verification effort. These examples, however, do not constitute a complete analysis of the entire code.

Example problems sets were solved by IGSM [Montgomery Watson, 1993] and benchmarked against solutions from MODFLOW [McDonald and Harbaugh, 1988], a code known to converge to the specified boundary value problems. The three problem sets included (1) groundwater flow with specified-head and head-dependent boundary conditions, (2) groundwater flow with drain boundary conditions, (3) coupled groundwater and surface water flow.

In all but problem set 1 with specified-head boundaries, IGSM solutions generally deviate significantly from those of MODFLOW. Example solutions under relatively mild forcing (e.g., pumping and changes in boundary conditions with time) display errors significant enough to undermine the validity of IGSM-based models. In other applications, errors may be either greater than or less than those displayed in the example problems. Significant temporal and spatial variability in hydrologic conditions on monthly time scales may mask errors in IGSM solutions. The potential for errors in IGSM solutions raises the concern that, in some cases, efforts to minimize such phenomena through adjusting model parameters may have been mistaken for calibration.
1.3 Principle Recommendations

The findings of this report show that several key algorithms used in IGSM differ substantially from the standard, tested methods employed by mainstream groundwater modeling codes. Furthermore, we demonstrate that, as a direct consequence of these non-standard features, IGSM model results can contain significant errors for typically encountered hydrologic conditions, both at the local and regional scales. The errors are not necessarily significant for every IGSM implementation but can be expected for a wide variety of systems. These shortcomings are further exacerbated by the fact that IGSM users will often be unable to anticipate the hydrologic conditions under which the errors will manifest.

Options to resolve this problem include either (1) fixing IGSM or (2) using an alternative model. This report discusses the pros and cons of each alternative. Alternatives to upgrading IGSM include modifying IGSM or modifying an established groundwater flow code. Application of an alternative model was the only available option at the time of this report.
2. INTRODUCTION

This report summarizes findings of a review of the Integrated Groundwater and Surface-Water Model (IGSM) [Montgomery Watson, 1993] conducted at the University of California, Davis (UCD) Hydrologic Sciences Graduate Group at the request of the California Water and Environmental Modeling Forum (CWEMF). The scope of this report includes a review and analysis of the theory of IGSM and the IGSM code, as well as several example problems.

Dr. Young S. Yoon began development of the three-dimensional, finite-element based Integrated Groundwater and Surface-Water Model (IGSM) in 1976 at the University of California, Los Angeles [Montgomery Watson, 1993]. Originally designed to simulate confined groundwater flow, IGSM subsequently underwent major revisions for use in various projects, including California’s Central Valley Groundwater Surface Water Model (CVGSM) developed for the United States Bureau of Reclamation, California State Department of Water Resources, California State Water Resources Control Board and Contra Costa Water District [Montgomery Watson, 1993]. Additional modifications were made during application of IGSM to the Salinas Valley Groundwater Basin.

In recent years, predictions from the Integrated Groundwater and Surface-Water Model (IGSM) have guided the planning and management of California’s water resources. The CVGSM played a key role in the Programmatic Environmental Impact Statement of the Central Valley Project Improvement Act (CVPIA) and the popularity of IGSM that followed this application. Applications of IGSM include, but are not limited to, Sacramento County, Pajaro Valley, Friant Service Area (San Joaquin Valley), Alameda County, City of Sacramento, Pomona Valley, Salinas Valley, Chino Basin, American River Watershed Service Area, Imperial Valley, and the Western San Joaquin Valley [Montgomery Watson, 1993]. The model has also been applied in groundwater basins in Colorado and Florida [Montgomery Watson, 1993]. Today, use by federal, state and local governmental agencies in nearly every major basin in California make IGSM the most widely used groundwater model for managing water resources of the State.

The IGSM code has changed little since its application to the CVGSM. Conceptually, it appears similar to more widely used groundwater models, such as MODFLOW [McDonald and
Harbaugh, 1988]. IGSM incorporates simple stream, land-use, and empirical vadose-zone models with inflows and outflows linked to a central groundwater model that includes a choice of boundary conditions (Fig. 2.1). Additional linkages between model components route flows within the system, e.g., between stream and land-use components. The stream model is similar to that of MODFLOW 2000 [Prudic, 1989]. Land-use and empirical vadose-zone models compute net percolation (recharge) to the groundwater system. IGSM requires unconfined conditions in the uppermost layer.

This report summarizes the outcome of a formal review of IGSM and includes (1) overview and analysis of the theory and code, (2) example problems, and (3) conclusions and recommendations.

3. REVIEW OF THEORY

This review of theory is limited to the basic streamflow and groundwater flow solution methodology. Many of the conclusions from our analysis, however, apply to additional features of the code not covered in this review. The notation used in this report differs from that of the Integrated Groundwater and Surface Water Model Documentation and User Manual [Montgomery Watson, 1993] to facilitate a more general discussion of the theory.

![Diagram of IGSM hydrologic components](image-url)

**Figure 2.1.** Interaction between hydrologic components of IGSM [after figure 2-2 of Montgomery Watson, 1993].
3.1 Governing Equations

3.1.1 Groundwater Flow
IGSM divides the domain into discrete layers. The vertically discrete governing three-
dimensional groundwater flow equation for layer $k$ is expressed as [Montgomery Watson, 1993]

$$
S_k \frac{\partial h_k}{\partial t} = \frac{\partial}{\partial x} \left[ T_k \left( h_k \right) \frac{\partial h_k}{\partial x} \right] + \frac{\partial}{\partial y} \left[ T_k \left( h_k \right) \frac{\partial h_k}{\partial y} \right] - L_{k-1} \left( h_k, h_{k-1} \right) \left( h_k - h_{k-1} \right) - L_k \left( h_k, h_{k+1} \right) \left( h_k - h_{k+1} \right) + F_i \left( h_k \right) \delta \left( x - x_i \right), \forall k
$$

(3.1.1)

$$
h(x, t) = f(x, t) \text{ on } \Gamma_1
$$

(3.1.2)

$$
T \mathbf{n} \cdot \nabla h_k = g(x, t) \text{ on } \Gamma_2
$$

(3.1.3)

$$
h(x, 0) = h_0(x)
$$

(3.1.4)

where the storage coefficient $S$ is defined as

$$
S_k = \begin{cases} 
    b_k S_s & \text{for confined flow} \\
    S_y & \text{for unconfined flow}
\end{cases}
$$

(3.1.4)

$b(x,t)$ is saturated thickness $[L]$, $S_s(x)$ is specific storage $[L^{-1}]$, $S_y$ is specific yield, $h(x,t)$ is hydraulic head $[L]$, $T(x,t)=K(x)b(x,t)$ is transmissivity $[L^2T^{-1}]$, $K(x)$ is isotropic horizontal hydraulic conductivity $[LT^{-1}]$, $L_k = \overline{K_v}^{k+1}_{k} / d_{k,k+1}$ is vertical leackance $[T^{-1}]$ between layer $k$ and $k+1$ (potentially a function of head), $b(x,t)$ is saturated thickness $[L]$, $S_s(x)$ is specific storage $[L^{-1}]$, $S_y$ is specific yield, $h(x,t)$ is hydraulic head $[L]$, $T(x,t)=K(x)b(x,t)$ is transmissivity $[L^2T^{-1}]$, $K(x)$ is isotropic horizontal hydraulic conductivity $[LT^{-1}]$, $L_k = \overline{K_v}^{k+1}_{k} / d_{k,k+1}$ is vertical leackance $[T^{-1}]$ between layer $k$ and $k+1$ (potentially a function of head), $\overline{K_v}^{k+1}_{k}$ is average vertical $K$ $[LT^{-1}]$ between layers $k$ and $k+1$, $d_{k,k+1}$ is the vertical distance $[L]$ between midpoints of the layer, $F_i$ is a source $[L^3T^{-1}]$ at location $x_i$, $\delta$ is the Dirac delta function $[L^{-2}]$ (a function that applies $F_i$ to locations $x_i$), $\mathbf{n}$ is a unit vector normal to the boundary of the domain $\Gamma$, and $f[L]$, $g[L^2T^{-1}]$, and $h_0[L]$ are known functions. If the head in layer $k$ falls below the surface $u_k(x)$, the system becomes unconfined at that point and the saturated thickness $b$, and therefore vertical leackance and transmissivity, change with hydraulic head because $b(x,t)=h(x,t)-u_k(x)$, where $u_{k+1}(x)$ is the elevation of the surface between layers $k$ and $k+1$. Note that the governing equations given on page 2-12 of Montgomery Watson [1993] are expressed for confined flow only. The equations expressed here are more general and account for variations in transmissivity with saturated thickness. Such
changes are considered within the IGSM code as discussed on page 2-17 of Montgomery Watson [1993].

It is important to recognize that equation (3.1.1) is nonlinear. That is, the solution for $h$ depends on parameters $(T, S)$ and $F$ values that are themselves dependent on $h$. Additional nonlinearities enter via the boundary conditions. In order for the numerical approximation of (3.1.1) to consistently represent the physics underlying (3.1.1) (or, for the numerical solution to “converge” onto equation (3.1.1)), special solution techniques must be used to deal with the nonlinearities.

### 3.1.2 Streams

The stream model of IGSM is similar to that of MODFLOW 2000 [Prudic, 1989]. The mass-balance equation governing flow along a stream reach at any instant in time equates the change in flow per unit length of stream to the inflow minus losses to groundwater. Considering a coordinate system $\xi$ aligned with the stream, then this word equation can be expressed mathematically as

$$
\frac{dQ_s}{d\xi} = Q_i \delta(\xi - \xi_i) d\xi - Q_s \delta(\xi - \xi_R) d\xi
$$

where $Q_s$ is flow in the stream [L$^3$T$^{-1}$], $Q_i$ are inflows [L$^3$T$^{-1}$] at locations $\xi_i$, $Q_s$ is the flow lost or gained [L$^3$T$^{-1}$] from the stream to the aquifer at locations $\xi_R$, and $\delta$ [L$^{-1}$] is the Dirac delta function. Equation 3.1.5 is the governing differential equation for streamflow routing with no storage and is the basis for the more familiar discrete numerical approximation to this equation used in MODFLOW and IGSM (see Section 3.2.2 Numerical Approximation of the Equations Governing Streamflow). Head [L] in the stream $h_R$ is computed from a streamflow rating curve, a function of flow rate and channel geometry [Montgomery Watson, 1993]. Heads in the stream establish head-dependent boundary conditions for the groundwater model as described in the following section.
3.1.3 Head-Dependent Boundary Conditions

Head-dependent boundaries in IGSM include general-head, stream and drain boundaries. For example, consider the head-dependent boundary condition that couples stream and aquifer models. Here we have

\[ Q_R = \begin{cases} C_R (h_R - h), & h \geq z_R \\ C_R (h_R - z_R), & h < z_R \end{cases} \]  

(3.1.6)

where \( C_R = K_R P_w L_R / b_R \) is streambed conductance \([L^2 T^{-1}]\), \( K_R \) is streambed hydraulic conductivity \([LT^{-1}]\), \( P_w \) is width (or wetted perimeter) \([L]\) of stream channel segment, \( L_R \) is length \([L]\) along the stream channel segment, \( b_R \) is bed thickness \([L]\), \( h_R \) is the head \([L]\) in the stream, \( h \) is hydraulic head \([L]\) in the aquifer, and \( z_R \) is the elevation \([L]\) of the bottom of the streambed.

IGSM attempts to solve equations (3.1.1) and (3.1.5) coupled through boundary condition (3.1.6). IGSM incorporates an estimate for \( Q_R \) as a source-sink term in \( F \) of the governing equation rather than incorporating equation (3.1.6) into the full mass balance calculation, as is the standard procedure in all other codes such as MODFLOW. This is accomplished through an explicit formulation of 3.1.6. This issue is discussed further below.

3.2 Solution Methodology

Here we consider the solution methodology of IGSM. Figure 3.2.1 [Montgomery Watson, 1993] illustrates the general flow of the IGSM code. IGSM solves explicitly for streamflow and semi-implicitly for groundwater flow, in sequence, with the groundwater flow solution limited to monthly time steps. The explicit solution for streamflow uses the hydraulic head solution of the previous month. The semi-explicit solution for groundwater flow uses the head solution of the previous month, but only in certain terms of the finite element equations. After describing this solution methodology in more detail, we contrast it with the quasilinearization approach used in MODFLOW.

---

1 IGSM contains a parameter KOPTST within the code that controls how head gradient terms in the stream aquifer interaction are computed. All simulations are performed with KOPTST = 1, which yields (like MODFLOW) a gradient calculation based on atmospheric pressure (unsaturated conditions) at the bottom of the stream bed when the average hydraulic head is below the elevation of the bottom of the streambed.
Figure 3.2.1. The flow of the IGSM code [from Montgomery Watson, 1993].
3.2.1 Numerical Approximation of the Groundwater Flow Equations

Discretizing space and applying the Galerkin finite-element technique on equation (3.1.1) yields the system of equations for all layers $k$ (in indicial notation) [Montgomery Watson, 1993]

$$D_{ij} \frac{\partial h_j}{\partial t} = G_{ij}(h) h_j + F_i(h)$$  \hspace{1cm} (3.2.1)

where $G$ is the conductance matrix \([L^2T^{-1}]\) and also includes vertical leakance terms, $D$ is the capacitance matrix \([L^2]\), and $h$ is the hydraulic head \([L]\) at discrete nodal locations. As a function of $h$, $F$ can include flow due to head-dependent boundary conditions (e.g., general head condition), not just simple sources and sinks. The functional dependence of $G$ on $h$ arises from the unconfined flow approximation in the top layer of the model.

The way IGSM approximates the spatial derivatives within the time step is very important to the analysis presented in this work, as shown below. Of concern are the terms that are a function of hydraulic head in equation 3.2.1. The usual method for solving equation 3.2.1 is to approximate the time derivative using finite differences and linearize the time discrete approximations using an existing estimate of $h^i$ in some terms. IGSM approximates the spatial derivatives and head-dependent source/sink term as follows:

$$D_{ij} \left( h_j^i - h_j^{i-\Delta t} \right) = G_{ij}(h^{i-\Delta t}) \left[ \Theta h_j^i + (1 - \Theta) h_j^{i-\Delta t} \right] + F_i(h^{i-\Delta t})$$  \hspace{1cm} (3.2.2)

where values of $\theta$ between 0 and 1 can be chosen by the user. Rearranging (3.2.2) yields the linear system of equations

$$A_{ij}(h^{i-\Delta t}) h_j^i = B_i(h^{i-\Delta t}) + F_i(h^{i-\Delta t})$$  \hspace{1cm} (3.2.3)

where $A = \Delta t^{-1} D - \Theta G$ and $B = [\Delta t^{-1} D + (1-\Theta)G] h^{i-\Delta t}$. For all $\Theta \neq 0$, e.g., $\Theta = 0.5$ or $\Theta = 1$, IGSM implements a semi-implicit (or semi-explicit) scheme, linearizing the problem by approximating the matrix $A$ using the head at the previous time step $h^{i-\Delta t}$. Similarly, the right-hand side of (3.2.3) appears as an explicit function of $h^{i-\Delta t}$, without consideration for the possibility that $F$ could be formulated implicitly for linear functions of $h^i$ while maintaining linearity in the system of equations, for example, when implementing head-dependent boundary conditions as considered in Section 3.2.3. In other words, through an explicit formulation, IGSM neglects changes (within a time step) in $h$ as they affect the properties ($A$ and $B$) and certain boundary
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conditions. This is not a common way of approximating the spatial derivatives and head-dependent source/sink term. More commonly, $\mathbf{F}$ and $\mathbf{A}$ are formulated implicitly using values of $h^i$ that are more closely related to the final values, as discussed in section 3.3.

The vertically discrete representation of the governing equations effectively yields a finite difference approximation to the spatial derivatives in the vertical. IGSM is quasi-three dimensional because of the added degree of freedom to adjust vertical leakance independent of the vertical conductivity of adjacent layers.

3.2.2 Numerical Approximation of the Equations Governing Streamflow

Discretizing space in equation (3.1.5) yields the familiar equality at steady state between the flow at the downstream node $i + 1$, $Q_{s_{in}}$, with the sum of the flow at upstream node $i$, $Q_{s_i}$, known net inflow from sources $Q_{i_s}$, and computed net flow lost to or gained from the aquifer system $Q_{r_i}$:

$$Q_{s_{in}} = Q_{s_i} + \sum_j Q_{i_j} - Q_{r_i} \quad (3.2.4)$$

Equation (4) is solved for all nodes $i$. The head in the stream $h_{R_i}$ is computed from $Q_{s_i}$ via streamflow rating curves. Flow lost to or gained from the aquifer system $Q_{r_i}$ is a function of the head in the stream through a head-dependent boundary condition as considered in the following section.

3.2.3 Implementing Head-Dependent Boundary Conditions

IGSM solves a discrete finite difference approximation of the streamflow equation (3.2.4) at time $t$, linearized using the hydraulic head of the previous time step $h^{i-\Delta t}$ to compute losses to the aquifer system at a given nodal location as

$$Q_{r_i}' = \begin{cases} C_R (h_{R_i}' - h^{i-\Delta t}_R), & h^{i-\Delta t}_R > z_R \\ C_R (h_{R_i}' - z_R), & h^{i-\Delta t}_R < z_R \end{cases} \quad (3.2.5)$$

Thus the head-dependent stream boundary condition for $h^{i-\Delta t} > z_R$ enters equation (3.2.3) in $\mathbf{F}$ as a specified flux:

$$A_j (h^{i-\Delta t}_j) h_j' = B_j (h^{i-\Delta t}_j) + C_R_{ij} (h_{R_j}' - h^{i-\Delta t}_j) \quad (3.2.6)$$
where the matrix $C_R$ contains the streambed conductance on the diagonal at locations that correspond with stream nodes and zeros elsewhere. Note that the flux depends on the hydraulic head of the previous time step and the head in the stream at the current time step. IGSM implements other head dependent conditions similarly. In contrast, the standard way of implementing head-dependent boundary conditions is to incorporate the equation directly into the matrix system represented by $A$ and $B$ in equation (3.2.3). In this method, the term $h_j^{t+\Delta t}$ in (3.2.6) is replaced by $h_j^t$, i.e., is not assumed, but rather is computed implicitly, thereby more fully accounting for the dynamic connection between the groundwater system and external phenomena such as streams and drains.

3.2.4 Time Step

IGSM operates on fixed monthly time steps (stress periods) $\Delta t$. Consequently, specified boundary conditions and sources are monthly. Time and length units are English and fixed due to internal unit conversions. Note that an option is available to simulate the streamflow equation (3.1.5) on a daily time step. This choice, however, does not affect the monthly time step associated with solution of the groundwater flow equations. Rather, the daily time step option accumulates a total seepage over the month for input into the right hand side of the numerical approximations to the groundwater flow equations.

3.2.5 Solution of the Linearized Approximation

IGSM solves equation (3.2.6) via block successive over relaxation [Montgomery Watson, 1993]. Optional solvers are not available for IGSM.

3.2.6 Computing and Reporting Budgets

IGSM computes budgets at time $t$ for head-dependent boundary sources of water to the groundwater system, e.g., stream losses or gains, from the solution $h^{t+\Delta t}$ of the previous time step. This accounting method is consistent with the semi-explicit (in time) solution methodology, but (at any given time) does not consider the dependence of boundary fluxes on the solution for hydraulic head.
3.2.7 Convergence

IGSM does not incorporate methods, such as quasilinearization, to converge on the governing nonlinear boundary value problem. Instead, the nonlinear problem is linearized and this linearized problem is solved once for each time step. Model output, to the screen during execution, showing sequential convergence to a solution actually shows convergence of the linear solver to this linearized problem only, not convergence of the entire solution.

3.3 Comparison with Established Solution Techniques

Here we compare the solution methodology of IGSM with the established technique of quasilinearization used by many other codes, for example MODFLOW. Bellman and Kalaba [1965] pioneered quasilinearization as a technique to solve nonlinear boundary-value problems such as those of the coupled equations in (3.1.1), (3.1.4) and (3.1.6). With this technique, we quasilinearize equation (3.2.1) to yield [Willis and Yeh, 1987]

\[ D_{ij} \frac{dh_{ij}^m}{dt} = G_{ij} \left( h_{ij}^{m-1} \right) h_{ij}^m + F_i \left( h^m \right) \]  (3.3.1)

where \( h_{ij}^{m-1} \) is known, \( h_{ij}^m \) is supposed to appear linearly in \( Q \), and \( m \) denotes the iteration step in the quasilinearization solution algorithm. Again, \( F \) here can represents flow due to head-dependent boundary conditions (e.g., general head condition). The quasilinearized form of the streamflow equation (3.2.4) is written as

\[ Q_{5i+1}^{i,m} = Q_{5i}^{i,m} + \sum_j Q_{ij}^i - Q_{Ri}^{i,m-1} \]  (3.3.2)

where \( Q_{Ri}^{i,m-1} \) is given for any node \( i \) as

\[ Q_{Ri}^{i,m-1} = \begin{cases} C_{Ri} \left( h_{Ri}^{i,m} - h_{i}^{i,m-1} \right), & h_{i}^{i,m-1} > z_R \\ C_{Ri} \left( h_{Ri}^{i,m} - z_R \right), & h_{i}^{i,m-1} < z_R \end{cases} \]  (3.3.3)

When incorporating the head-dependent boundary condition of relationship in 3.3.3 into equation 3.3.1, MODFLOW uses \( h_{i}^{i,m} \) in place of \( h_{i}^{i,m-1} \). Substituting relationship 3.3.3 (for \( h_{i}^{i,m} > z_R \)) into \( F \) of 3.3.1 yields

\[ A_{ij} \left( h_{j}^{i,m-1} \right) h_{ij}^{i,m} = B_i \left( h_{i}^{i,m-1} \right) + C_{ji} \left( h_{Rj}^{i,m} - h_{j}^{i,m} \right) \]  (3.3.4)

Finally, rearranging (3.3.4) yields the linear system of equations...
\[
\left[A_j(h_{j,m-1}) + C_{R_{ij}}\right]h_{j,m} = \left[B_i(\Delta h_{t-m}) + C_{R_{ij}} h_{R_{ij}}\right] \tag{3.3.5}
\]

The quasilinearization algorithm proceeds with an initial guess for \(h_0\), commonly chosen as \(h_{t-\Delta t}\). Equation (3.3.2) is then solved via finite differences for \(Q_{s,t,1}\) with \(h_{s,t,1}\) computed from rating curves. Finally equation (3.3.5) is solved for \(h_{t,1}\), \(m\) is incremented and the process is repeated until a convergence criteria, normally \(|h_{i,m} - h_{i,m-1}| < \varepsilon, \forall i\), is satisfied. Upon convergence within these “outer iterations,” stream budgets can be computed by incrementing \(m\) and solving (3.3.2) with (3.3.3) using \(h_{s,t,m-1}\), instead of \(h_{s,t,m}\), to ensure consistency between boundary conditions of the stream and groundwater system.

Standard techniques take advantage of the linear dependence on hydraulic head of the head-dependent boundary flux in the solution (rearranging Eq. 3.3.4 to yield Eq. 3.3.5). Further, application of the quasilinearization technique addresses convergence of groundwater and stream model solutions (as well as other coupled models, e.g., land use) with a user specified tolerance \(|h_{i,m} - h_{i,m-1}| < \varepsilon, \forall i\) that controls numerical accuracy of the solution to the nonlinear boundary value problem. The IGSM algorithm lacks these features, resulting in groundwater, stream, and land use model solutions that are inconsistent with one another (i.e., not “converged”).

### 3.4 Stability and Accuracy of the Numerical Approximations

As equation (3.2.2) suggests, IGSM contains an option to adjust time derivative approximations by adjusting \(\Theta\). The specific choice of time derivative approximation affects stability and accuracy of IGSM model solutions. In addition, stability and accuracy depend on both time step and spatial discretization. As the time step of IGSM is fixed, choice of spatial discretization controls accuracy of solutions.
stable. The Crank Nicholson approximation is normally preferred over the fully implicit approximation ($\Theta = 1$) as it is of higher order accuracy [Willis and Yeh, 1987].

Results shown in Appendix A, however, demonstrate the peculiar conditional stability (or lack of stability) of IGSM implicit approximations. Conditional stability results from lagging the head in time (in the computation of an effective transmissivity) to linearize the unconfined flow problem. Since IGSM does not implement quasilinearization to converge the solution, time lagging the head means that the computation of transmissivity in the unconfined flow problem is always accomplished explicitly (i.e., using heads from the previous time step). Hence, we refer to the IGSM algorithm as semi-implicit (or semi-explicit). The conditionally stable semi-implicit Crank Nicholson approximation ($\Theta = 0.5$) of IGSM apparently imposes severe restrictions on spatial discretization, comparable to an explicit approximation (see Appendix A). In contrast, solutions appear relatively stable for $\Theta = 1$, as we will show for example, when comparing results in Appendix A to those of example problem set 1. Thus, we suggest that for practical purposes the IGSM option of changing $\Theta$ to any value other than $\Theta = 1$ is ineffectual. Therefore, all simulations performed within this report use the semi-implicit approximation ($\Theta = 1$), which appears considerably more stable than the semi-implicit Crank Nicholson. The following section considers critical node spacing associated with the fixed monthly time step of IGSM.

### 3.4.2 Accuracy and Spatial Discretization

Spatial discretization and time step can also affect accuracy of a numerical solution. The finite element approximations converge in the limit as the node spacing and time step simultaneously go to zero. Anderson and Woessner (p. 205; 1992) reference De Marsily (1986) in suggesting an order of magnitude estimate for maximum initial time step to ensure accuracy of the model solution for a uniform node spacing. This critical time step $\Delta t_c$ is given as

$$\Delta t_c = S \Delta x^2 / 4T$$

(3.4.1)

which is computed from the elemental Fourier Number, defined as the product of the total conductance of an element and the critical time step divided by the capacitance. Total conductance is simply the summation of all conductances representing connections to the subject node. Equation (3.4.1) is equivalent to the stability criteria for an explicit algorithm. It is common practice to increase time step sequentially from the critical time step as the solution
progresses in time under constant stresses (i.e., during a stress period) and to establish a new critical time step when stresses change significantly. The ultimate goal is avoid inaccuracies arising from temporal discretization of the governing equations at a given spatial resolution. Inaccuracies resulting from coarse discretization of the spatial domain can only be controlled by refining node spacing.

Equation (3.4.1) suggests a relationship between time step and node spacing. Like the critical time step for a given node spacing, there exists a critical node spacing for a given time step. Since the time step of IGSM is fixed, accuracy can only be controlled through spatial discretization. Thus, critical node spacing can be key to a reliable solution. In two dimensions, and for a regular node spacing ($\Delta x = \Delta y$), the critical time step criteria of Eq. (3.4.1) can be rearranged to yield a minimum critical nodal spacing $\Delta x_c$ for a fixed time step $\Delta t$

$$\Delta x_c = \sqrt{\frac{4T\Delta t}{S}}$$  (3.4.2)

Tables 3.4.1 and 3.4.2 show critical node spacing for typical ranges of conductivity and aquifer thickness (Table 3.4.1), and specific storage (Table 3.4.2) for $\Delta t = 30$ days (~1 month). Note that for confined flow and the simplifying assumptions considered herein, the critical node spacing is independent of the saturated thickness.

The critical node spacing represents an order of magnitude estimate for the minimum node spacing necessary for accuracy of the model solution at a corresponding time step $\Delta t$. It does not, however, address all inaccuracies arising from coarse discretization of the model domain. Normally one chooses a node spacing that meets the demands of a particular application, both in accuracy and in required resolution of the solution. Therein lies a potentially serious dilemma for IGSM and its users: Satisfying the critical node spacing may require a grid that is too coarse to achieve accuracy in the finite element approximations and/or too coarse to be of practical value for a particular application. The solution is to resolve the element mesh and time step simultaneously, but IGSM does not allow for the latter. As a result, the critical node spacing (associated with the fixed monthly time step) given in Tables 3.4.1 and 3.4.2 can impose impractical restrictions on node spacing leading to error prone solutions. In real-world applications, such errors may be important, but are likely to be overlooked because (1) an accurate solution with which to compare is not generally available without application of robust
model and (2) errors may be masked by fluctuations in head due to temporal and spatial variability in sources, sinks and boundary conditions.
**Table 3.4.1**
Minimum Critical Node Spacing $\Delta x_c$ (ft) for IGSM Solution (Unconfined Flow)*

<table>
<thead>
<tr>
<th>Saturated Conductivity (ft/day)</th>
<th>0.1</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>22</td>
<td>69</td>
<td>219</td>
<td>693</td>
</tr>
<tr>
<td>10</td>
<td>22</td>
<td>69</td>
<td>219</td>
<td>693</td>
<td>2191</td>
</tr>
<tr>
<td>100</td>
<td>69</td>
<td>219</td>
<td>693</td>
<td>2191</td>
<td>6928</td>
</tr>
<tr>
<td>1000</td>
<td>219</td>
<td>693</td>
<td>2191</td>
<td>6928</td>
<td>21909</td>
</tr>
</tbody>
</table>

* $S_y = 0.25$

**Table 3.4.2**
Minimum Critical Node Spacing $\Delta x_c$ (ft) for IGSM Solution (Confined Flow)

<table>
<thead>
<tr>
<th>Specific Conductivity (ft/day)</th>
<th>0.1</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-3}$</td>
<td>110</td>
<td>346</td>
<td>1095</td>
<td>3464</td>
<td>10954</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>346</td>
<td>1095</td>
<td>3464</td>
<td>10954</td>
<td>34641</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>1095</td>
<td>3464</td>
<td>10954</td>
<td>34641</td>
<td>109545</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>3464</td>
<td>10954</td>
<td>34641</td>
<td>109545</td>
<td>346410</td>
</tr>
</tbody>
</table>
### 3.4.3 Convergence and water-balance error reporting

A well-known method for evaluating convergence of the numerical solution is to examine the model water-balance error. IGSM does not compute a true water-balance error. The water-balance error is normally determined from a mass balance on the model domain computed from the current model solution for hydraulic head. Unfortunately, the information provided by IGSM to the user at any time does not include a mass balance computed from the current model solution (see section 3.2.6 Computing and Reporting Budgets), but rather, explicitly from results of the previous time step. Therefore, the water balance will appear practically error free (within the convergence tolerance of the linear equation solver) as sources and sinks are specified explicitly. As a result, there is no way for a user of IGSM to readily assess convergence of the model solution from reported water budgets. Because the IGSM user manual [Montgomery, Watson, 1993] lacks a detailed description of how to interpret IGSM water budgets and these budgets will typically appear virtually error free, the unsuspecting user can be misled into thinking that the corresponding numerical solution balances mass.

### 3.5 FORTRAN Code and Model Documentation

A review of the IGSM code found that it is unstructured and includes undocumented additions. Well structured code includes liberal use of comments, appropriate indentation and modules that make a code easy to follow and upgrade. The IGSM code could be improved in each of these areas.

An independent review team [Tariq Kadir of California Department of Water Resources, personal communication 2002] recently confirmed our general findings of undocumented additions to the code. Appendix B considers one such example in which *ad hoc* methods, relating to head-dependent boundaries are incorporated in an apparent attempt to stabilize the semi-implicit approximation used in IGSM. When implemented properly, as described in section 3.3 of this report (also see the section on general head boundary conditions in McDonald and Harbaugh, 1988), head-dependent boundaries do not require additional code of this type to
stabilize solutions. We did not seek to identify all undocumented additions. Nevertheless, the example in Appendix B lacks sound theoretical basis.

Formal documentation for IGSM appears limited to the Integrated Groundwater and Surface Water Model Documentation and User Manual [Montgomery Watson, 1993]. As discussed above, in many cases, this manual lacks the detail necessary to understand the underlying theory and the computations performed in the IGSM code. Further, the User’s Manual lacks a description of the code, i.e., its arrays, variables, and logic. The code has more than 17,000 lines [Tariq Kadir of California Department of Water Resources, personal communication 2002], lacks structure (comments, indentation, appropriate modularity) making it difficult to understand. Efforts are currently underway by DWR to resolve this problem [Tariq Kadir of California Department of Water Resources, personal communication 2002]. Finally, the IGSM User’s Manual contains no verification problems or examples to demonstrate that the code correctly solves the governing boundary value problem.

4. EXAMPLE PROBLEMS

Example problems described herein are simple and represent common hydrologic phenomena. Problems were chosen based upon review of the code to demonstrate some of the known issues that would need to be resolved prior to conducting a more comprehensive verification effort. The example problems considered are solved with IGSM [Montgomery Watson, 1993] and benchmarked against solutions from MODFLOW [McDonald and Harbaugh, 1988], a code known to converge to the specified boundary value problems. Correspondence between nodal locations of the IGSM finite element mesh and MODFLOW finite difference grid used in all example problems is shown in Figure 4.1. The grids and boundary conditions are applied such that solutions computed at MODFLOW and IGSM nodes are comparable.

Example problems sets include (1) unconfined groundwater flow with specified-head and head-dependent boundary conditions, (2) groundwater flow with drain boundary conditions, (3) coupled groundwater and surface water flow. The problems are simple and aquifer parameters are within a typical range. In some instances, parameters are varied over a range of values to assess their effect on model error.
Additionally, the following is common for all example problems:

- In all verification problems, streamflow routing in IGSM is performed on a monthly time step, as we have selected a constant stream inflow within any given month.
- All simulations are performed with KOPTST = 1, such that gradient calculations in the computation of seepage losses from streams are consistent with those of MODFLOW and the theory discussed herein.
- All simulations, with the exception of those discussed in Appendix A, are performed with the semi-implicit option $\Theta = 1$, as the IGSM semi-implicit Crank Nicholson approximation, $\Theta = 1/2$, was found in most cases to be unstable.
- All MODFLOW simulations are performed with 25 time steps per month with a 1.1 time step multiplier (time steps are not constant; the time step multiplier specifies the rate at which time step length increases during the month; see McDonald and Harbaugh, [1988]).

IGSM and MODFLOW model input and output files are provided on the CD accompanying this report.
4.1 Problem Set 1: Unconfined Flow and Head-Dependent Boundary Conditions

Problem Set 1 tests the IGSM solution to unconfined groundwater flow, as well as its formulation and implementation of constant-head and head-dependent boundary conditions. Standard solution methods discussed in section 3.3 exploit the linearity of the head-dependent boundary condition, folding the condition directly into the linear system of equations solved at each time step (or multiple times per time step if quasilinearization is used). As discussed in section 3.2.3, IGSM computes the head-dependent boundary flux in any given month from solution of the hydraulic head in the previous month, effectively disconnecting the boundary condition from the groundwater system at any point in time. Problem 1 explores the consequences of this approach.

4.1.1 Boundary-Value Problem

The governing groundwater flow equation is given as

\[
S \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( Kh \frac{\partial h}{\partial x} \right)
\]

\[\text{initially, } h(x,0) = h_0 \]

\[h(x_1, t) = h_1 \]

\[h(x_2, t) = h_2 \]

Initially, the water table is \( h_0 = h_2 \). At the start of the simulation, the water table on the left boundary at \( x_1 \) is instantly lowered to \( h_1 \) and held constant for all \( t > 0 \).

4.1.2 Simulation

Figure 4.1.1 shows the single-layer MODFLOW computational grid. Although the domain is two dimensional, flow in one of the dimensions is zero (i.e., there is no variation in hydraulic head in that dimension) so as to render the problem effectively one-dimensional. Table 4.1.1 summarizes the corresponding parameters, boundary conditions, and initial conditions. Problems 1a and 1b as well as problems 1c and 1d, only differ by their node spacing of 1000 ft and 100 ft, respectively. Simulations include maintenance of boundary heads through either constant-head (Problems 1a and 1b) or head-dependent boundary conditions (Problems 1c and 1d).
latter, we specify a conductance of $K h \Delta y / (1 \text{ ft}) \ [\text{ft}^2\text{day}^{-1}]$, a value theoretically large enough to approximate the constant-head boundary conditions of the specified boundary-value problem.

### 4.1.3 Results and Discussion

Figures 4.1.2 and 4.1.3 compare IGSM and MODFLOW solutions at months 1 and 12. Solutions compare well. The explicit approximation of the transmissivity $T$ explains differences in the solutions for $h(x)$, that are greater for Problem 1b due to the finer node spacing than 1a. Differences are greatest in month 1, and eventually, after several months of holding boundary conditions steady, the IGSM solution converges to the MODFLOW solution. Alternative parameter values will yield more or less deviation from the more accurate MODFLOW solution.
Figure 4.1.2. IGSM and MODFLOW solutions for hydraulic head (ft) plotted against distance in the $x$-direction at months 1 and 12 for problem 1a, specified head boundary conditions with $\Delta x = 100$ ft. MODFLOW and IGSM solutions compare poorly in month 1. Solutions in month 12 are nearly identical.

Figure 4.1.3. IGSM and MODFLOW solutions for hydraulic head (ft) plotted against distance in the $x$-direction at months 1 and 12 for problem 1b, specified head boundary conditions with $\Delta x = 1000$ ft. MODFLOW and IGSM solutions are nearly identical.
In many cases, such differences may not constitute a significant source of error. However, since typical basins can experience wide fluctuations in hydraulic head on monthly time scales, errors associated with the explicit approximation of $T$ may be significant in some cases, depending on the specific application.

Figures 4.1.4 and 4.1.5 compare IGSM and MODFLOW solutions for boundary conditions maintained with head-dependent (“general-head”) boundaries. The IGSM solution deviates significantly from that of MODFLOW demonstrating that the head-dependent boundary condition of IGSM is error prone. As described in the Section 3 of this report, “head-dependent” boundary conditions in IGSM are implemented as specified fluxes calculated explicitly based on heads at the previous time step, rather than as true head-dependent boundary conditions. Further, ad hoc additions to the code potentially affect head-dependent boundary flux calculations in the code (see Appendix B).

![Figure 4.1.4. IGSM and MODFLOW solutions for hydraulic head (ft) plotted against distance in the x-direction at months 1 and 12 for problem 1c, general-head boundary conditions with $\Delta x = 100$ ft. IGSM solutions deviate significantly from those of MODFLOW.](image)
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Investigation of Methods Used in the IGSM

Figure 4.1.5. IGSM and MODFLOW solutions for hydraulic head (ft) plotted against distance in the x-direction at months 1 and 12 for problem 1d, general-head boundary conditions with $\Delta x = 1000$ ft. IGSM solutions deviate significantly from those of MODFLOW.

4.2 Problem Set 2: Drain Boundary Conditions

Problem set 2 simulates groundwater flow in an agricultural setting in response to pumping, flow to tile drains and recharge. Recall that standard methods exploit linearity of the head-dependent (drain) boundary condition, folding the condition directly into the system of equations solved at each time step (or multiple times per time step if quasilinearization is used). IGSM, on the other hand, computes the boundary flux at any given month based on solution of the hydraulic head from the previous month. Problem 2 explores the IGSM solution approach to drain boundary conditions.

4.2.1 Boundary-Value Problem

The governing groundwater flow equation is given as

$$
S_y \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( Kh \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( Kh \frac{\partial h}{\partial y} \right) + Q_w(t) \delta(x - x_w, y - y_w) + Q_d(h) \delta(x - x_d, y - y_d)
$$

(4.2.1)

$$
h(x, y, 0) = h_0
$$

(4.2.2)

$$
h(x, y, t) = h_i
$$

(4.2.3)
Investigation of Methods Used in the IGSM

\[
h(x, y, t) = h_2
\]  \hspace{1cm} (4.2.4)

\[
\partial h(x, y, t)/\partial y = 0
\]  \hspace{1cm} (4.2.5)

\[
\partial h(x, y, t)/\partial y = 0
\]  \hspace{1cm} (4.2.6)

\[
Q_D = \begin{cases} 
C_D \left[ z_D - h(x_D, y_D, t) \right], & h > z_D \\
0, & h < z_D 
\end{cases}
\]  \hspace{1cm} (4.2.7)

where \(Q_w\) is volumetric flux of sources or sinks [L\(^3\)T\(^{-1}\)] due to recharge (net percolation) or pumpage at locations \(x=x_w\) and \(y=y_w\), \(Q_D\) is volumetric flux [L\(^3\)T\(^{-1}\)] to tile drains at locations \(x=x_D\) and \(y=y_D\), \(z_D\) is the elevation [L] of the drain, \(C_D\) is the drain-boundary conductance [L\(^2\)T\(^{-1}\)], \(x_1\), \(x_2\), \(y_1\) and \(y_2\) correspond to the extent of the domain in the \(x\)- and \(y\)-direction, \(\delta\) [L\(^{-2}\)] is the Dirac delta function. Boundary and initial conditional are \(h_0 = h_1 = h_2\). The drain boundary conductance represents the geometric and hydraulic conductivity terms governing hydraulic connection between the aquifer and the drain (i.e., everything in Darcy’s equation except for the head drop).

4.2.2 Simulation

Table 4.2.1 summarizes corresponding parameters, boundary conditions, and initial conditions for problems 2a – 2c. Specified IGSM parameters for boundary area (BA) and boundary distance (BD) yield the tabulated values of drain conductance \(C_D\). Figure 4.2.1 illustrates the location of tile drains, recharge and wells. Recharge is simulated with “wells” in IGSM and the recharge package in MODFLOW. Table 4.2.2 summarizes pumping and recharge schedules. Note that the three problems differ only in specified drain conductance, \(C_D\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2a</th>
<th>2b</th>
<th>2c</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h_0, h_1, h_2) (ft)</td>
<td>195.0</td>
<td>195.0</td>
<td>195.0</td>
</tr>
<tr>
<td>(K) (ft/day)</td>
<td>20.0</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>(S_y)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>(\Delta x, \Delta y) (ft)</td>
<td>1000.0</td>
<td>1000.0</td>
<td>1000.0</td>
</tr>
<tr>
<td>(BH, z_D) (ft)</td>
<td>192.0</td>
<td>192.0</td>
<td>192.0</td>
</tr>
<tr>
<td>(BA)* (ft(^3))</td>
<td>(10^6)</td>
<td>(10^5)</td>
<td>(10^4)</td>
</tr>
<tr>
<td>(BD)** (ft)</td>
<td>50.0</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>(C_D) (ft(^2)/day)</td>
<td>(4 \times 10^5)</td>
<td>(4 \times 10^4)</td>
<td>(4 \times 10^3)</td>
</tr>
<tr>
<td>Elevation of Base (ft)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

* IGSM drain elevation is specified as BH.
** IGSM computes conductance from specified boundary area (BA) and boundary distance (BD) as \(C_D = KBA/BD\), where \(K\) is equal to \(K\) of the boundary node.
Figure 4.2.1. MODFLOW finite difference grid of problem set 2.

Table 4.2.2: Pumpage and Recharge of Problems 3a – 3d

<table>
<thead>
<tr>
<th>Month</th>
<th>Pumpage* (gpm)</th>
<th>Recharge** (ft/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.02592</td>
</tr>
<tr>
<td>2</td>
<td>2452</td>
<td>0.02592</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.02592</td>
</tr>
<tr>
<td>4</td>
<td>2452</td>
<td>0.02592</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>0.02592</td>
</tr>
<tr>
<td>6</td>
<td>2452</td>
<td>0.02592</td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>0.02592</td>
</tr>
<tr>
<td>8</td>
<td>2452</td>
<td>0.02592</td>
</tr>
<tr>
<td>9</td>
<td>0.0</td>
<td>0.02592</td>
</tr>
<tr>
<td>10</td>
<td>2452</td>
<td>0.02592</td>
</tr>
<tr>
<td>11</td>
<td>0.0</td>
<td>0.02592</td>
</tr>
<tr>
<td>12</td>
<td>2452</td>
<td>0.02592</td>
</tr>
</tbody>
</table>

* Distributed equally to 4 model cells  
** Applied to the 10 nodes with tile drains.
4.2.3 Results and Discussion

Figures 4.2.2 – 4.2.4 compare IGSM and MODFLOW solutions for drain flow versus time for problems 2a – 2c, respectively. IGSM solutions deviate significantly from those of MODFLOW because IGSM fails to properly execute head-dependent boundaries. Two important aspects of the IGSM solutions are apparent from the plots: First, oscillations in IGSM solutions are out of phase with those from MODFLOW because IGSM computes flow to drains explicitly from the hydraulic head solution of the previous month, rather than considering the solution at the current month. Second, and perhaps more important, IGSM solutions for drainflow are insensitive to changes in drain boundary parameters, i.e., IGSM solutions in Figures 4.2.2 – 4.2.4 are all the same.

Figure 4.2.2. IGSM and MODFLOW solutions for flow to drains plotted against time for problem 2a, $C_D = 4 \times 10^5 \text{ ft}^2/\text{day}$. IGSM solutions deviate significantly from those of MODFLOW. Values plotted for every time step of the MODFLOW solution show temporal fluctuations out of phase with those of IGSM. The IGSM solution is the same as in Fig. 4.2.3 and 4.2.4, i.e., IGSM drainflows are incorrectly independent of drain-boundary parameters.
LaBolle, Ahmed, and Fogg  Hydrologic Sciences, U.C. Davis

**Figure 4.2.3.** IGSM and MODFLOW solutions for flow to drains plotted against time for problem 2b, $C_D = 4 \times 10^4 \text{ ft}^2/\text{day}$. IGSM solutions deviate significantly from those of MODFLOW. Values plotted for every time step of the MODFLOW solution show temporal fluctuations out of phase with those of IGSM. The IGSM solution is the same as in Fig. 4.2.2 and 4.2.4, i.e., IGSM drainflows are incorrectly independent of drain-boundary parameters.

**Figure 4.2.4.** IGSM and MODFLOW solutions for flow to drains plotted against time for problem 2c, $C_D = 4 \times 10^3 \text{ ft}^2/\text{day}$. IGSM solutions deviate significantly from those of MODFLOW. Values plotted for every time step of the MODFLOW solution show temporal fluctuations out of phase with those of IGSM. The IGSM solution is the same as in Fig. 4.2.2 and 4.2.3, i.e., IGSM drainflows are incorrectly independent of drain-boundary parameters.
As discussed in Appendix B, undocumented additions to the code can limit the computed drain flow (as well as losses from streams). One such addition controls IGSM computed drainflow in problems 2a - 2c. This addition was also included in previous versions of the code, for example, the code used in the CVGSM model. Here IGSM computes flow to drains as

\[
Q'_D = \begin{cases} 
\Delta t^{-1}A_i S_i \left[ z_D - h_i^{t-\Delta t} \right], & h > z_D \\
0, & h < z_D
\end{cases}
\]  

(4.2.8)

where \(A_i\) is the planar area \([L^2]\) associated with node \(i\). Comparing with Eq. 4.2.7, one can see that IGSM is not solving the specified boundary value problem. It appears that Eq. (4.2.8) is an ad hoc measure intended to correct errors in the drain boundary condition. Further, 4.2.8 lacks dependence on drain boundary parameters, which explains why all IGSM solutions are the same when the drain boundary conductance is varied over two orders of magnitude from problem 2a to 2c. Even if one inactivated relationship 4.2.8 in the code (see Appendix C, Section C.2.2.5, response to comment #23 on p. 72), errors would still arise from improper implementation of the head-dependent, drain boundary condition.

Figure 4.2.5 compares computed total flow to the drains from IGSM and MODFLOW over the 12 month period. Note again that IGSM solutions do not change with drain boundary conductance. Therefore, as in problem 2b, for example, long-time average results may deviate significantly from the correct solution, demonstrating that problems with IGSM can ultimately lead to severe water budget errors that are not necessarily remedied by averaging over longer time periods. Similar errors can be expected in problems where models (groundwater, land use and/or stream) are coupled, for example where diversions from streams become applied water in the land-use model or where aquifer stream interaction is considered. Here mass-balance errors arise due to (1) the boundary conditions that couple the models and (2) the inability of IGSM to simultaneously converge these coupled models. The following verification problem with aquifer stream interaction demonstrates these issues.

4.3 Problem Set 3: Aquifer-Stream Interaction

Problem Set 3 tests the IGSM solution of coupled groundwater and surface water flow. As the name IGSM implies, the code was developed with such problems in mind. Standard solution
methods (e.g., those used in MODFLOW) approach this nonlinear boundary value problem with techniques aimed at simultaneously converging models of groundwater and streamflow coupled through head-dependent boundary conditions. IGSM, on the other hand, solves explicitly for streamflow and then groundwater flow, in sequence, with the groundwater flow solution limited to monthly time steps. Problem set 3 explores the ability of the IGSM approach to simulate coupled groundwater and surface water flow.

Due to its explicit formulation and lack of a methodology to simultaneously converge the groundwater and surface water models, one can expect, a priori, IGSM solutions to deviate from those of standard techniques, with the greatest deviations occurring in cases where the groundwater and surface water are highly coupled. The streambed conductance controls the degree of coupling between groundwater and surface water in the model. Therefore, specified streambed conductance is varied over a range to demonstrate its affect on model solutions.

Figure 4.2.5. IGSM and MODFLOW solutions for total flow to drains over 12 months for problem 2a-2c. IGSM solutions deviate significantly from those of MODFLOW.
4.3.1 Boundary-Value Problem

The governing groundwater flow equation is given as

\[ S \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( K_h \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_h \frac{\partial h}{\partial y} \right) + Q_w(t) \delta(x-x_w, y-y_w) + Q_h(h) \delta(x-x_R, y-y_R) \]  

\[ h(x,y,0) = h_0 \]  

\[ h(x, y, t) = h_i \]  

\[ h(x, y, t) = h_2 \]  

\[ \frac{\partial h(x, y, t)}{\partial y} = 0 \]  

\[ \frac{\partial h(x, y, t)}{\partial y} = 0 \]  

The equation governing streamflow is given as

\[ \frac{dQ_s}{d\xi} = Q_i \delta(\xi - \xi_i) - Q_r \delta(\xi - \xi_r) \]  

\[ Q_s = \begin{cases} C_R (h_R - h), & h > z_R \\ C_R (h_R - z_R), & h < z_R \end{cases} \]  

Initially, the water table is \( h_0 = h_1 = h_2 \).

4.3.2 Simulation

Figure 4.3.1 illustrates the single-layer MODFLOW and IGSM computational grids. Table 4.3.1 summarizes model parameters, boundary conditions, and initial conditions for problems 3a – 3c. The three problems differ by the specified streambed conductance \( C_R \), which varies over three orders of magnitude. Streambed conductance \( C_R \) in Table 4.3.1 is expressed as \( \frac{C_R}{P_w L_R} \) day\(^{-1}\) = \( \frac{K_R}{b_R} \) day\(^{-1}\), where \( P_w \) is effective channel width [L], \( L_R \) is the length of the stream reach [L], \( K_R \) is the conductivity [LT\(^{-1}\)] of the streambed, and \( b_R \) is the thickness of the streambed [L]. Table 4.3.2 provides the specified streamflow rating curve (modeled after rating curves used in the CVGSM IGSM application). A modified MODFLOW stream package developed to incorporate such rating curves is used to compare solutions of MODFLOW with those of IGSM. Table 4.3.3 summarizes specified monthly pumping and stream inflow for scenarios 1 - 3. Scenarios 1 and 2 include steady stream inflow of 500 and 5000 cfs, respectively, with no pumpage, and test IGSM under a simple condition in which groundwater and stream models are coupled. Scenario 3 describes three separate simulations that specify steady stream inflow of 100 cfs with steady
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Figure 4.3.1. MODFLOW finite difference grid of problem set 3.

Table 4.3.1: Parameters of Problems 3a – 3c

<table>
<thead>
<tr>
<th>Parameter</th>
<th>3a</th>
<th>3b</th>
<th>3c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_0, h_1, h_2$ (ft)</td>
<td>200.0</td>
<td>200.0</td>
<td>200.0</td>
</tr>
<tr>
<td>$K$ (ft/day)</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$S_y$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Delta x, \Delta y$ (ft)</td>
<td>1000.0</td>
<td>1000.0</td>
<td>1000.0</td>
</tr>
<tr>
<td>$C_R/P_w L_R$ (day$^{-1}$)</td>
<td>1.0</td>
<td>10.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Elevation of Base (ft)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4.3.2: Stream Rating Table for Problems 3a – 3c

<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>Width (ft)</th>
<th>Flow Rate (cfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>1000</td>
</tr>
<tr>
<td>15</td>
<td>250</td>
<td>5000</td>
</tr>
<tr>
<td>20</td>
<td>300</td>
<td>20000</td>
</tr>
</tbody>
</table>
Table 4.3.3: Stream Inflow and Pumpage for Scenarios 1 – 3, Problem Set 3

<table>
<thead>
<tr>
<th>Month</th>
<th>Scenario 1 Inflow (cfs)</th>
<th>Scenario 2 Inflow (cfs)</th>
<th>Pumpage* (cfs)</th>
<th>Scenario 3 Inflow (cfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500.0</td>
<td>5000.0</td>
<td>0.0</td>
<td>100.0</td>
</tr>
<tr>
<td>2</td>
<td>500.0</td>
<td>5000.0</td>
<td>0.0</td>
<td>100.0</td>
</tr>
<tr>
<td>3</td>
<td>500.0</td>
<td>5000.0</td>
<td>0.0</td>
<td>100.0</td>
</tr>
<tr>
<td>4</td>
<td>500.0</td>
<td>5000.0</td>
<td>0.0</td>
<td>100.0</td>
</tr>
<tr>
<td>5</td>
<td>500.0</td>
<td>5000.0</td>
<td>10.0-50.0</td>
<td>100.0</td>
</tr>
<tr>
<td>6</td>
<td>500.0</td>
<td>5000.0</td>
<td>10.0-50.0</td>
<td>100.0</td>
</tr>
<tr>
<td>7</td>
<td>500.0</td>
<td>5000.0</td>
<td>10.0-50.0</td>
<td>100.0</td>
</tr>
<tr>
<td>8</td>
<td>500.0</td>
<td>5000.0</td>
<td>10.0-50.0</td>
<td>100.0</td>
</tr>
<tr>
<td>9</td>
<td>500.0</td>
<td>5000.0</td>
<td>10.0-50.0</td>
<td>100.0</td>
</tr>
<tr>
<td>10</td>
<td>500.0</td>
<td>5000.0</td>
<td>10.0-50.0</td>
<td>100.0</td>
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<tr>
<td>11</td>
<td>500.0</td>
<td>5000.0</td>
<td>10.0-50.0</td>
<td>100.0</td>
</tr>
<tr>
<td>12</td>
<td>500.0</td>
<td>5000.0</td>
<td>10.0-50.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

* Distributed equally to 6 model cells

pumpage of 10, 25 and 50 cfs, respectively, beginning in month 5. In this scenario, pumping lowers hydraulic head inducing losses from the stream. Problems 3a – 3c are each run for scenarios 1 and 2. Only problem 3b is run for scenario 3.

4.3.3 Results and Discussion

Figures 4.3.2a – 4.3.4a (scenario 1) and 4.3.2b – 4.3.4b (scenario 2) compare IGSM and MODFLOW solutions for streamflow (at the node corresponding to the last reach of the stream) versus time for problems 3a –3c, respectively. Figures show results from both scenarios 1 and 2. IGSM solutions generally deviate significantly from those of MODFLOW. IGSM results for problem 3a (Fig. 4.3.2a and b) display instabilities and inaccuracies that appear to grow with time for scenario 1, despite steady conditions. The degree of instability and inaccuracy in problem 3a (Fig. 4.3.2a and b) is significant, but relatively small compared to that of problems 3b and 3c (Figs. 4.3.3a and b and 4.3.4a and b), in which there is greater connection between groundwater and surface water. Thus, the degree of instability and inaccuracy in IGSM solutions increases with the degree of coupling, controlled by streambed conductance $C_R$ between groundwater and surface water. Results also show greater relative errors in scenario 1 than in scenario 2, i.e., relative errors are greatest in low flow situations. Similarly, Figures 4.3.5a – 4.3.7a (scenario 1) and 4.3.5b – 4.3.7b (scenario 2) compare IGSM and MODFLOW solutions for hydraulic head versus time (at the node corresponding to the end of the stream) for problems 3a –3c, respectively. Solutions from IGSM and MODFLOW deviate significantly, with the former again showing a trend of increasing instability and inaccuracy with increasing streambed conductance and/or decreasing streamflow.
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Figure 4.3.2a. IGSM and MODFLOW solutions for streamflow plotted against time for problem 3a, $C_R/P_wL_R = 1.0$ day$^{-1}$, scenario 1, steady stream inflow of 500 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW and errors manifest as oscillations increasing with time under steady inflow conditions.

Figure 4.3.2b. IGSM and MODFLOW solutions for streamflow plotted against time for problem 3a, $C_R/P_wL_R = 1.0$ day$^{-1}$, scenario 2, steady stream inflow of 5000 cfs, no pumpage. IGSM solutions display notable, but small errors manifesting as oscillations under steady inflow conditions.
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Figure 4.3.3a. IGSM and MODFLOW solutions for streamflow plotted against time for problem 3b, $C_R/P_w L_R = 10.0$ day$^{-1}$, scenario 1, steady stream inflow of 500 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW with severe errors that manifest as oscillations, even under steady inflow conditions. Note that errors under low-flow conditions (scenario 1) are significantly greater than those for higher flows (scenario 2) as shown in Fig. 4.2.5.

Figure 4.3.3b. IGSM and MODFLOW solutions for streamflow plotted against time for problem 3b, $C_R/P_w L_R = 10.0$ day$^{-1}$, scenario 2, steady stream inflow of 5000 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW with severe errors that manifest as oscillations, even under steady inflow conditions.
Figure 4.3.4a. IGSM and MODFLOW solutions for streamflow plotted against time for problem 3c, $C_R/P_wL_R = 100.0 \text{ day}^{-1}$, scenario 1 steady stream inflow of 500 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW.

Figure 4.3.4b. IGSM and MODFLOW solutions for streamflow plotted against time for problem 3c, $C_R/P_wL_R = 100.0 \text{ day}^{-1}$, scenario 2, steady stream inflow of 5000 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW and errors oscillation severe.
Figure 4.3.5a. IGSM and MODFLOW solutions for hydraulic head (ft) plotted against time for problem 3a, $C_R/P_wL_R = 1.0 \text{ day}^{-1}$, scenario 1, steady stream inflow of 500 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW and errors manifest as oscillations increasing with time under steady inflow conditions.

Figure 4.3.5b. IGSM and MODFLOW solutions for hydraulic head plotted against time for problem 3a, $C_R/P_wL_R = 1.0 \text{ day}^{-1}$, scenario 2, steady stream inflow of 5000 cfs, no pumpage. IGSM solutions display notable, but small errors manifesting as oscillations under steady inflow conditions.
4.3.6a. IGSM and MODFLOW solutions for streamflow plotted against time for problem 3b, $C_R/P_nL_R = 10.0 \, \text{day}^{-1}$, scenario 1, steady stream inflow of 500 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW with severe errors that manifest as oscillations, even under steady inflow conditions. Note that errors under low-flow conditions (scenario 1) are significantly greater than those for higher flows (scenario 2) as shown in Fig. 4.2.5.

4.3.6b. IGSM and MODFLOW solutions for hydraulic head plotted against time for problem 3b, $C_R/P_nL_R = 10.0 \, \text{day}^{-1}$, scenario 2, steady stream inflow of 5000 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW with severe errors that manifest as oscillations, even under steady inflow conditions.
Figure 4.3.7a. IGSM and MODFLOW solutions for hydraulic head plotted against time for problem 3c, $C_R/P_{wL_R} = 100.0 \text{ day}^{-1}$, scenario 1 steady stream inflow of 500 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW.

Figure 4.3.7b. IGSM and MODFLOW solutions for hydraulic head plotted against time for problem 3c, $C_R/P_{wL_R} = 100.0 \text{ day}^{-1}$, scenario 2, steady stream inflow of 5000 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW and errors oscillation severe.
Figure 4.3.8 compares IGSM and MODFLOW solutions for streamflow (at the node corresponding to the last reach of the stream) versus time for problem 3b, scenario 3. Again solutions deviate significantly. An additional anomaly resulting from the IGSM solution methodology is also apparent in the results plotted in Fig. 4.3.8: The solution for streamflow in any given month is insensitive to the groundwater conditions in that month. For example, streamflow of the IGSM solution in month 5 is insensitive to pumpage in month 5. In other words, one could pump as much water as the aquifer will yield in any given month without affecting streamflow in that month. This problem arises because of the sequential nature of the IGSM algorithm in which equations for streamflow in a given month are solved prior to, and independently of, the groundwater flow solution in that month. In effect, at any given time, groundwater and surface water models are decoupled.

Figure 4.3.8. IGSM and MODFLOW solutions for streamflow plotted against time for problem 3b, $C_R/P_w L_R = 10.0$ day$^{-1}$, scenario 3, steady stream inflow of 100 cfs, pumpage beginning in month 5 of 10 cfs and 25 cfs. IGSM solutions deviate significantly from those of MODFLOW. The IGSM solution for streamflow in any given month is insensitive to the groundwater conditions in that month as illustrated by the complete lack of response to pumpage in month 5.
The IGSM solution algorithm fails to solve the coupled groundwater and surface water models. The degree of inaccuracy and error increases with increasing coupling, i.e., increasing streambed conductance $C_R$, between groundwater and surface water systems. The unsuspecting user may interpret such errors as valid solutions to the governing boundary value problem and seek to minimize instability and oscillations during calibration through decreasing streambed conductance. Further, temporal variability in hydrologic conditions, which commonly change significantly on monthly time scales, may tend to mask such errors in IGSM solutions.

5. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 General Solution methodology

IGSM uses a semi-implicit method to linearize the finite-element and finite-difference approximations of (1a) and (3). When active, many of the additional features (i.e., features not tested herein) of IGSM, that would otherwise introduce similar nonlinearities in the approximation, are also linearized using this explicit methodology. Generally, accuracy with such explicit approximations requires that the state of the system change slowly in time; the time step is decreased to achieve accuracy and stability. IGSM incorporates a fixed monthly time step, thereby affording the user little control over convergence and accuracy.

Standard solution methods invoking linearization (e.g., those used in MODFLOW) generally do so in the context of a quasilinearization scheme. Quasilinearization is a proven, iterative method for the solution of nonlinear boundary value problems. IGSM lacks such a method to achieve convergence.

IGSM incorporates ad hoc methods that can help to stabilize solutions. One example, related to implementation of head-dependent boundaries, is provided in Appendix B. This undocumented addition to the code lacks a rigorous theoretical foundation.

IGSM does not provide correct information on local or global water balance errors that would routinely be used by modelers to detect and diagnose numerical inaccuracies stemming from incomplete convergence, or numerical inaccuracies.
5.2 Aquifer Stream Interaction and Convergence of Coupled Models

IGSM lacks a method to achieve simultaneous convergence of coupled models, including groundwater and streamflow models. The IGSM algorithm does not attempt to achieve convergence of the IGSM solution to the specified nonlinear boundary value problem.

5.3 Head-Dependent Boundary Conditions

IGSM does not include true, head-dependent boundary conditions. In IGSM so-called head-dependent boundary conditions, including general head, stream, and drain conditions, are implemented as a specified flux based upon the known head of the boundary node from the previous time step. Yet the solution methodology lacks a technique to converge to the proper boundary flux. Thus one should expect significant errors, and possibly oscillatory behavior, in the solution when implementing general-head, stream or drain boundaries, except when computed heads change very slowly in time.

5.4 Critical Node Spacing

The critical node spacing developed herein provides a rough (probably order of magnitude estimate) for minimum node spacing to maintain accuracy based on the fixed monthly time step of IGSM. Tabulated values for critical node spacing highlight the dilemma faced by the IGSM user. The fixed monthly time step imposes restrictions on critical node spacing that can be greater than the node spacing required for accuracy (1) by the finite element approximations and/or (2) by the practical requirements for resolution of the solution.

5.5 Review of the Code and Documentation

To our knowledge, formal documentation for IGSM is limited to the Integrated Groundwater and Surface Water Model Documentation and User Manual [Montgomery Watson, 1993]. Our review found that in some instances the manual lacks sufficient detail on both underlying theory and how the theory is implemented in the code. Similarly, the code contains undocumented additions lacking a rigorous theoretical foundation. The IGSM User’s Manual lacks a detailed description of the code, its arrays, variables, and logic. IGSM contains more than 17,000 lines of unstructured (i.e., difficult to read and modify or improve) code. In the IGSM User’s Manual
there are no verification problems and examples that demonstrate a working model (one that solves the governing boundary value problem).

5.6 Example Problems Results

Example problems sets were solved by IGSM [Montgomery Watson, 1993] and benchmarked against solutions from MODFLOW [McDonald and Harbaugh, 1988], a code known to converge to the specified boundary value problems. The three problems sets included (1) unconfined groundwater flow with specified-head and head-dependent boundary conditions, (2) groundwater flow with drain boundary conditions, (3) coupled groundwater and surface water flow. In all but problem set 1 with specified-head boundaries, IGSM solutions generally deviate significantly from those of MODFLOW. Examples demonstrate the consequences of the non-standard implementation methods that were identified during the review process. These methods lead directly to errors and instability. Depending on the specific applications, such errors may be either greater than or less than those displayed in the example problems herein. However, example solutions, under relatively mild forcing, display inaccuracies significant enough to question the validity of IGSM-based model applications in general. Temporal variability in hydrologic conditions, which commonly change significantly on monthly time scales, may tend to mask such errors in IGSM solutions. The potential for instability and errors in IGSM solutions raises significant concerns that some IGSM model-calibration efforts have mistaken minimizing such phenomena for model calibration.

5.7 Recommendations

The findings of this report show that IGSM model results can contain significant errors. Options to resolve this problem include either (1) fixing IGSM or (2) using an alternative model. This section discusses the pros and cons of each alternative. Alternatives to upgrading IGSM include modifying IGSM or modifying an established groundwater flow code. It is our opinion that application of an alternative model is the best option available to solve the types of problems that practitioners are attempting to solve with IGSM.

5.7.1 Fixing IGSM

The error prone solutions to simple example problems, a code development process apparently lacking effective verification efforts, and the likelihood of additional problems not identified
herein are drawbacks to fixing IGSM. Perhaps the most compelling is the weight of clearly evident, error-prone model results in some simple example problems. In turn, these errors appear to be the byproduct of a somewhat ad hoc code development process that lacked effective verification efforts. Inadequate verification suggests additional problems with the code, not identified herein. This, together with the sheer length of the code (comprised of more than 17,000 lines) and the undocumented additions make reliably upgrading and verifying IGSM a potentially long and difficult process. Such an effort, if undertaken, should include documentation, analysis, and verification of all aspects of the code.

5.7.2 Build a new IGSM through Upgrading an Alternative Model Code

Upgrading an alternative model code to directly solve problems posed to IGSM is a possible remedy. If this approach is pursued, we recommend that such a code be modular in construction (such that it can be easily upgraded), non-proprietary, well documented and well supported by the greater hydrologic community.

5.7.3 Use of an Alternative Model

The problems currently considered by IGSM can be solved with alternative modeling platforms that are known to be reliable. Table 5.7.1 compares IGSM with MODFLOW 2000, a tested groundwater code with typical features.
Table 5.7.1: Comparison of IGSM 5.0 and MODFLOW 2000 Code Attributes

<table>
<thead>
<tr>
<th>CODE ATTRIBUTE</th>
<th>IGSM 5.0</th>
<th>MODFLOW 2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three dimensional</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Confined flow</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Unconfined flow</td>
<td>Yes*</td>
<td>Yes</td>
</tr>
<tr>
<td>Unsaturated flow</td>
<td>Yes**</td>
<td>Yes (full version to be released)</td>
</tr>
<tr>
<td>Spatial derivative approximation</td>
<td>Finite Element</td>
<td>Finite Difference</td>
</tr>
<tr>
<td>Time derivative approximation</td>
<td>Semi Explicit</td>
<td>Implicit/Quasilinearization</td>
</tr>
<tr>
<td>Variable time step</td>
<td>No, Fixed Monthly</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Boundary Conditions**

<table>
<thead>
<tr>
<th>Flux (pumpage/recharge)</th>
<th>Yes</th>
<th>Yes</th>
</tr>
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<tbody>
<tr>
<td>Specified head</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>General head</td>
<td>Yes*</td>
<td>Yes</td>
</tr>
<tr>
<td>Tile drains</td>
<td>Yes*</td>
<td>Yes</td>
</tr>
<tr>
<td>Evaporation</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Additional Features**

<table>
<thead>
<tr>
<th>Streamflow routing</th>
<th>Yes*</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lakes/Reservoirs</td>
<td>Yes**</td>
<td>Yes</td>
</tr>
<tr>
<td>Subsidence</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Landuse (surface budget accounting)</td>
<td>Yes**</td>
<td>No (must calculate external to code)</td>
</tr>
<tr>
<td>Parameter estimation</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Particle tracking</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Choice of solvers</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>GUI</td>
<td>No</td>
<td>Yes (USGS through ARGUS 1 and third party)</td>
</tr>
<tr>
<td>Companion Transport Code</td>
<td>No</td>
<td>MOC3D, PATH3D, MODPATH, MT3D, etc.</td>
</tr>
<tr>
<td>Third party support</td>
<td>Limited</td>
<td>Extensive</td>
</tr>
<tr>
<td>Public agency support</td>
<td>No</td>
<td>Extensive, USGS</td>
</tr>
</tbody>
</table>

*Tested herein and shown to be error prone

**Known to be error prone based on implementation in code and findings of this review.
6. REFERENCES


APPENDIX A:

THE IGSM SEMI-IMPLICIT CRANK NICHOLSON APPROXIMATION

Equation 3.2.1 is nonlinear in $h$. The numerical approximation in Eq. 3.2.2 linearizes Eq. 3.2.1 by evaluating $G$ at $h^{t+\Delta t}$. For $\Theta = 0.5$ we refer to Eq. 3.2.2 as a semi-implicit Crank Nicholson approximation. Here we investigate the stability and accuracy of this approximation by benchmarking solutions from IGSM against MODFLOW, a code known to be stable under the conditions considered herein. Figures A.1 and A.2 compare MODFLOW and IGSM results for months 1 and 12 from test Problems 1a and 1b, respectively, for $\Theta = 0.5$. Results from the IGSM solutions deviate significantly from those of MODFLOW. Instability increases with decreasing $\Delta x$.

Figure A.1. MODFLOW and IGSM computed hydraulic head plotted against distance for problem 1a with $\Theta = 0.5$. 
Figure A.2. MODFLOW and IGSM computed hydraulic head plotted against distance for problem 1b with $\Theta = 0.5$. 
APPENDIX B:

UNDOCUMENTED ADDITIONS TO THE CODE

This Appendix describes one of many undocumented additions to the IGSM code. This particular undocumented addition limits the maximum drainflow to values computed using the following formula

\[ Q_D = \begin{cases} \frac{\Delta t}{A_i} S_i \left[ z_p - h_i - \Delta h \right] & h > z_p \ \\ 0 & h < z_p \end{cases} \]  

where \( A_i \) is the planar (x-y plane) area \([L^2]\) associated with node \( i \). Table B.1 compares values hydraulic head and drainflow computed by IGSM for node 4 problem 2a with values computed from Formula B.1 confirming that this undocumented addition is controlling in the case of problems 2a – 2c. The relationship in B.1 is based on an incomplete mass balance (for the volume associated with the drain boundary node) to assess the maximum amount of water that this node can take in a given time step. Components of the mass balance neglected include lateral and vertical groundwater flow and other sources and sinks applied at the node in a given time step. Further, the relationship lacks a drain-conductance term and is therefore not based on the physics governing flow to drains and insensitive to changes in drain-boundary parameters. Modifying the usual head-dependent drain boundary as per equation B.1 is unnecessary when drain boundaries are properly implemented (see McDonald and Harbaugh, 1988).
<table>
<thead>
<tr>
<th>Month</th>
<th>Drain Flow (ft³/month)</th>
<th>Drain Flow Computed Using Formula B.1 (ft³/month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150000</td>
<td>150000</td>
</tr>
<tr>
<td>2</td>
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</table>
APPENDIX C:
COMMENTS ON THIS REPORT AND RESPONSES TO COMMENTS

This appendix summarizes comments on this report, solicited by the CWEMF from a large group of IGSM users, and our responses to those comments. The section presents a summary of comments received; each comment is followed by a response. The complete text of all comments is available on the CD that accompanies this report.

Comments were received by Dr. Can Dogrul of California Department of Water Resources (CA DWR), Drs. Ali Taghavi and Saquib Najmus of WRIME, Inc., and Rich Juricich of the CA DWR. Comments from reviewers were generally helpful in improving this report.

C.1 Response to Comment of Dr. Can Dogrul, CA DWR

Dr. Can Dogrul of CA DWR voiced a single comment, calling to our attention an error in unit conversions of drainflow simulations in the original draft of this final document. We subsequently corrected this error and alerted other reviewers to the problem prior to receiving their comments.

C.2 Response to Comments of Drs. Ali Taghavi and Saquib Najmus of Water Resources and Information Management, Inc. (WRIME)

Drs. Ali Taghavi and Saquib Najmus of Water Resources and Information Management, Inc. (WRIME) authored 16 pages of comments. This section includes a summary of these comments and our responses, followed by detailed comments and responses. Comments from WRIME were helpful in improving the text of this report. In general, however, these reviewers’ comments regarding the technical content of this report are highly critical. Our review of these comments reveals a general lack of foundation for this criticism. In fact, our response below shows how many of these reviewers’ comments and simulation results substantially support our conclusions.

The previous work of Dr. LaBolle in the year 2000 included a discussion of many of the problems with IGSM that are now detailed in this report, including simulation examples of field-
scale stream aquifer interaction. The work by Dr. LaBolle was presented to Drs. Taghavi and Najmus of WRIME in a public forum at CA DWR in the summer of 2000. Thus, these reviewers were made aware of most of the problems discussed herein more than two years before the publication of this report. The code reviewed in this report was released by these reviewers as the latest update of IGSM and provided to us by CA DWR in late 2001. An update of the subroutine BOUND.FOR was provided as recently as January of 2002 by Dr. Saquib Najmus of WRIME to the California Water and Environmental Modeling Forum and then to us.

The work of Dr. LaBolle and others, as well as the work that culminated in this report, has led to the recognition of the problems with IGSM described herein. The California Department of Water Resources (CA DWR) has taken on the difficult task of producing new code that uses elements from IGSM, for example the same preprocessing code and linear solver. During this development process, CA DWR has been able to independently verify the test problems documented herein and reproduce results in this report (Tariq Kadir and Dr. Can Dogrul, CA DWR, personal communication).

The task assigned to reviewers was to evaluate the work presented in this report developed using IGSM 5.0. We were provided with code used by these reviewers and found that in their comments and analysis on drainflow, these reviewers have mistakenly used a version of IGSM modified to bypass the problem identified in Appendix B of this report. It is regrettable that this error has occurred. Nevertheless, use of the IGSM code modified to bypass a problem identified in this document suggests that it is recognized that (1) IGSM contains the problems identified in this report, (2) these problems compromise reliability of the code and its application, and (3) these problems are serious enough to warrant modification of the code.

C.2.1 Summary of Comments and Responses

C.2.1.1 Summary of Comments in “Section 1.0 Introduction” and Responses

- Reviewers suggest that the “report did not include any test cases that are representative of field conditions” when in fact, as our detailed response to comment #1 shows (see below), model parameters are within the range of those used in IGSM applications and
commonly found throughout California, and the hydrologic phenomena modeled are common to most IGSM applications.

- Reviewers suggest that the report does not establish a basis for using MODFLOW results as a benchmark, when this report clearly states that MODFLOW is known to converge to the specified boundary values problems.

- Reviewers suggest that “code changes to the standard versions of both IGSM and MODFLOW …undermines the objectivity and reliability of the review process,” when such changes were minor, and, in the case of IGSM, necessary to implement documented options in IGSM and upgrade from non-standard FORTRAN file OPEN statements in IGSM before recompiling the code.

- Reviewers imply that this report contains “incorrect assumptions about stream aquifer interactions and solution techniques used in IGSM” when in fact the report is consistent with the reviewers’ description of the code.

- Reviewers state that there are “errors/deficiencies in the IGSM problem set-ups and interpretations of the results” when our review and review by staff at CA DWR have found only one error, called to the attention of, and corrected prior to receiving comments from these reviewers.

- Reviewers suggest that “several conclusions in this Report are not well supported because of generalizations and extrapolations from a limited number of test problems,” when the fact is that the simple test problems in this review include hydrologic phenomenon common to most IGSM applications with parameters typical of those in California and, as such, should be solvable by a working code.

C.2.1.2 Summary of Comments in “3.0 Assumptions about IGSM” and Responses

- Reviewers assert that this report assumes that IGSM calculates stream aquifer interaction “on a monthly basis.” However, this report explains the IGSM option to use a daily sub-step and how this option differs little from the monthly option because the groundwater flow equations are solved on a monthly basis in each.

- Reviewers assume that this report contains conclusions regarding IGSM solution algorithms that are presumptuous because IGSM uses “a highly efficient numerical solution technique” known as the “method of fractional steps,” more commonly known as operator splitting. In fact, referred to herein as a sequential semi explicit algorithm,
this report discusses how it is that this method of fractional steps combined with a monthly time step is one source of error in IGSM solutions.

- Reviewers imply that stream and drain boundary conditions in IGSM and MODFLOW are different enough that solutions should not be compared to one another. Nevertheless, these boundary conditions are theoretically head-dependent boundary conditions in both codes.

C.2.1.3 Summary of Comments in “4.0 Formulation and Description of Test Problems” and Responses

- Reviewers’ comments suggest that the IGSM model components tested herein and the applicability of test problems to “real world situations” are not described in this report. Yet the theoretical aspects of the code being tested are discussed throughout this report and the report states that “aquifer parameters are within a typical range.”

- These reviewers suggest that this report does not discuss why the particular triangular finite element mesh was chosen, when this report states that: “The grids and boundary conditions are applied such that solutions computed at MODFLOW and IGSM nodes are comparable.”

C.2.1.4 Summary of Comments in “5.0 IGSM Model Set-up for Test Problems” and Responses

- Reviewers suggest that the code has been modified for “unspecified reasons” when, in fact, the need to implement certain options in IGSM requires modifying and recompiling the code.

- Reviewers suggest that code changes made as part of this review were not acknowledged in this report, when in fact they are specifically mentioned in this report and detailed on the companion CD which was provided to the reviewers.

- Reviewers state that use of the IGSM option of KOPTST = 1 is an error, when in fact it is the option with theoretical underpinnings comparable to that of MODFLOW as explained in section 3.2 Solution Methodology.

- Reviewers state that the addition of the statement in subroutine BOUND.FOR to extract drainflows is in error, when in fact our review and a review by staff at DWR found that this addition is correctly implemented.

- Reviewers used a version of BOUND.FOR code, modified to bypass a problem identified in Appendix B of this report, as a foundation for comments in Section 6. Thus,
differences in reviewers model runs and the model runs in this report are due, not to errors in this report, but to modifications to the IGSM code to specifically bypass a problem identified in this report.

- Reviewers indicate that initial conditions in test problem 3 are incorrect when in fact they are not, as verified by our review of this problem, and review by staff at DWR who are using this test problem in code development work.

C.2.1.5 Summary of Comments in “6.0 Interpretation of IGSM and MODFLOW Results” and Responses

- These reviewers state that “In order to understand the significance of the findings presented in the Report, we conducted additional model runs of selected test problems. The results of these model runs are used to interpret and explain the differences between IGSM and MODFLOW, as summarized below.” We were provided with the code used to make these additional runs. A review of this code has found that it has been modified (from its original version) to bypass the problem identified in Appendix B of this report.

- Reviewers suggest that test problems of general head boundaries (GHBs) were “not appropriate” and they run additional problems that they suggest show “reasonable simulation results” when in fact these new simulation results compare poorly with the correct results from MODFLOW.

- Reviewers plot drainflow results from modified code incorrectly, making IGSM results appear in phase with MODFLOW results, when in fact they are not.

- Reviewers run model simulations of aquifer stream interaction, changing parameters in an attempt to reduce instability with IGSM. Nevertheless, reviewers’ model results show large errors when compared with the correct MODFLOW results. Further, these reviewers suggest that changing stream inflow may be a possible remedy to reduce errors in results, a nonsensical suggestion since stream inflow is not a model input that one can simply adjust in real applications to reduce numerical errors.

- Reviewers suggest that MODFLOW results for aquifer stream interaction reach an instantaneous steady state, when in fact they equilibrate during the course of the first month.
C.2.1.6 Summary of Comments in “7. Validity of Conclusions of the Report” and Responses

- Reviewers suggest that “test problems … have no connection to a real world situation” when in fact, as our detailed response to comment #1 shows (see below) model parameters are within the range of those used in IGSM applications and commonly found throughout California. Importantly, the hydrologic phenomena modeled herein are common to most IGSM applications.

C.2.2 Detailed Comments and Responses

C.2.2.1 Response to key conclusions in the “1.0 Introduction”

1. “The Report demonstrates some limitations of the IGSM in regards of its applicability to certain set of theoretical problems being tested as part of the review process; however, the Report did not include any test cases that are representative of field conditions”

These reviewers apparently missed the following statement contained in the report: “The problems are simple and aquifer parameters are within a typical range. In some instances, parameters are varied over a range of values to assess their effect on model error.”

In fact, aquifer parameters and streamflow rates are within the range of those commonly encountered in real field conditions and in IGSM model applications. The following table compares test problem parameter ranges with those used in the IGSM application known as the Central Valley Groundwater and Surface Water Model (CVGSM) and shows that parameters of test problems are representative of those used in IGSM applications.

<table>
<thead>
<tr>
<th>Comparison of Test Problem Parameters with Parameters of CVGSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Horizontal K (ft/day)</td>
</tr>
<tr>
<td>Specific Yield</td>
</tr>
<tr>
<td>Streambed K (ft/day)</td>
</tr>
</tbody>
</table>

Earlier in our study we had voiced the desire to do a MODFLOW-IGSM comparison on the Sacramento County model, but the time and resources needed to do that were beyond the scope
of this project. Furthermore, it became clear to us during our tests of IGSM that we could produce regional-scale phenomena in our simple examples by manipulating the node spacing, as demonstrated in the report.

2. “The Report does not establish a basis for using the MODFLOW results as a benchmark, since MODFLOW is also an approximate numerical model. Therefore, the differences between IGSM and MODFLOW results should not necessarily be viewed as a measure of “errors” in IGSM.”

In the report we point out the excellent reliability of MODFLOW. Nevertheless, let us be more explicit: There is no other groundwater modeling code in existence that has been more extensively tested and more grounded in current theory for solving basic groundwater boundary value problems than MODFLOW. Not only has it undergone rigorous benchmarking (against known solutions) by the USGS before it’s release in 1988, but it has since been applied in hundreds (probably thousands) of groundwater problems during the last 15+ years while undergoing continuous scrutiny of the code. MODFLOW has withstood not only theoretical scrutiny by modeling experts worldwide, but also the test of repeated applications. The integrity and reliability of MODFLOW remains unequaled.

Now, we will reiterate numerical equivalencies between IGSM and MODFLOW. Since finite differences is a subset of finite elements, both IGSM and MODFLOW solutions should be approximately equal to each other, and in fact should correspond more closely to each other than to an analytical solution. In other words, finite element and finite difference solutions are practically identical on a rectilinear grid with triangular elements. This report states that “Example problems sets were solved by IGSM [Montgomery Watson, 1993] and benchmarked against solutions from MODFLOW [McDonald and Harbaugh, 1988], a code known to converge to the specified boundary value problems.” Further, this report goes on to state in Section 4 that “Correspondence between nodal locations of the IGSM finite element mesh and MODFLOW finite difference grid used in all example problems is shown in Figure 4.1. The grids and boundary conditions are applied such that solutions computed at MODFLOW and IGSM nodes are comparable.” These statements clearly establish a basis for using MODFLOW solutions as a benchmark.
3. “Reviewers made code changes to the standard versions both IGSM and MODFLOW (the reference model) in order to test example problems. This action undermines the objectivity and reliability of the review process.”

All changes to the IGSM and MODFLOW code are documented on the companion CD to this report. All changes are minor and necessary to test IGSM and benchmark it against MODFLOW. In fact, changes to IGSM were only necessary because implementation of some IGSM options, documented in the User’s Manual, require changing values in the source code itself and subsequently recompiling the IGSM code. Recompiling IGSM requires upgrades to the code because developers used non-standard FORTRAN coding which prevents compiling IGSM source code on all but Lahey-compatible compilers. Specific changes to IGSM and MODFLOW are repeated here for clarity:

- **IGSM:** Testing of aquifer-stream interaction required using the IGSM option of KOPTST=1. This option is documented in the IGSM User’s Manual and is implemented by changing this value from 0 to 1 in the source code. Changing the KOPTST value in the source code required recompiling IGSM. However, developers of the IGSM code used the nonstandard Lahey compiler extension “ACCESS=TRANSPARENT” in several OPEN statements. Recompiling IGSM required upgrading the code to ANSI-standard FORTRAN. Note also that binary files produced using “ACCESS=TRANSPARENT” are not standard, explaining why files produced with the standard FORTRAN version will not run with the version using non-standard FORTRAN.

- **MODFLOW:** The original version of the stream package in MODFLOW does not use rating curves to relate width, depth and flow in a stream. IGSM only uses rating curves. Therefore, to test aquifer-stream interaction in IGSM, we added a rating curve option in MODFLOW, making the codes directly comparable.
4. “The Report contains incorrect assumptions about stream aquifer interactions and solution techniques used in IGSM.” This comment is expanded on in Section 3.1 of these reviewer’s comments as: “The Report assumed that the stream aquifer interaction in IGSM is calculated on a monthly basis. This is not a correct assumption, as the IGSM uses a daily sub-step within a monthly time step to approximate the non-linear behavior of stream aquifer interactions. The groundwater head values from the previous time step is used as a starting value for iteration of daily sub-stepping within the current month. The groundwater head value below the stream node is updated at every daily sub-step by solving the mass balance equation in a finite volume. This updated groundwater head is used to calculate the stream gain and loss at the next daily sub-step within the monthly simulation time step.”

These reviewers are only partially correct regarding IGSM functionality: The daily sub-time-step in stream flow calculations is an option that can be implemented using DELTM = 0 (versus DELTM = 1 for monthly time steps in the stream flow package). This option is referred to in Section 3.2.4 of this report: “Note that an option is available to simulate the streamflow equation (3.1.5) on a daily time step. This choice, however, does not affect the monthly time step associated with solution of the groundwater flow equations. Rather, the daily time step option accumulates a total seepage over the month for input into the right hand side of the numerical approximations to the groundwater flow equations.”

These reviewers have identified one of the many “Undocumented additions to the code, lacking rigorous theoretical basis” referred to in item 7, Section 1.1 of this report. The reviewers refer to this as the “mass balance equation in a finite volume” used in streamflow calculations. It is for this reason that we documented (in Appendix B of this report) the nearly identical approach as implemented for drainflow calculations in IGSM. These so-called “mass balance” approaches, implemented within both the stream and drainflow elements of the code, fail to account for all fluxes within finite volumes (i.e., the elements associated with the stream or drain boundary node). In other words, mass is not balanced with these approaches. In the code, finite-volume mass-balance equations are implemented for both the daily and monthly stream-routing time stepping. As shown in Appendix B, such ad hoc attempts to stabilize the code are not a solution to the numerous problems identified with IGSM, but are in fact a part of the problem.
Finally, it was not within the scope of this review and evaluation of IGSM to document and test every feature of the code. Quoting from the introductory paragraph of this report: “These example problems by no means test all aspects of the code, but are adequate for assessing some of the foundational methods of IGSM.” This report’s detailed description of the IGSM stream package seeks to fill in many of the missing gaps left out of the IGSM documentation, explaining how boundary conditions and attempts at coupling stream and aquifer models are implemented therein. The report clearly explains how the poor performance and aberrant behavior of IGSM solutions to test problems stems from its improper formulation and faulty solution techniques.

5. **“There are errors/deficiencies in the IGSM problem set-ups and interpretations of the results”**

We have thoroughly reviewed our work, specifically with respect to comments of the reviewers, and we find no basis for such claims. Staff at the California Department of Water Resources have been using all example problems and results contained in this report as a part of an effort to verify IGSM2, a new code that is a rewrite of IGSM. Staff at DWR found only one error, that is, a problem with drainflow problems (of our original draft) called to our attention by Can Dogrul of DWR and detailed in the response to his comments. An Email was sent to Dr. Taghavi of WRIME explaining the details of this error significantly prior to receiving comments from WRIME. This error is not present in this version of the final report.

6. **“Several conclusions in the Report are not well supported because of generalizations and extrapolations from a limited number of test problems.”**

The few simple test problems in this review include hydrologic phenomenon common to all IGSM applications and should be solvable by a working code. Yet, quoting from the Section 1.0 Executive Summary of this report: “This review found IGSM to be unreliable for a number of simple example problems. These example problems by no means test all aspects of the code, but are adequate for assessing some of the foundational methods of IGSM. Results identify and elucidate some of the issues that would need to be solved prior to conducting a more extensive verification effort.” Further, quoting from Section 1.2 Example Problems Results: “Example solutions under relatively mild forcing (e.g., pumping and changes in boundary conditions with
time) display errors significant enough to undermine the validity of IGSM-based models. In other applications, errors may be either greater than or less than those displayed in the example problems. Significant temporal and spatial variability in hydrologic conditions on monthly time scales may mask errors in IGSM solutions.” Importantly, in applications to complex systems there is no way for a user of IGSM to test how poorly the model is doing, i.e., non-convergence to the specified boundary-value problem, without building a comparable model with a code that is known to converge for comparison. These reviewers’ comments are expanded on in Section 7. “Validity of Conclusions of the Report” and more detailed responses to these comments are provided in the following sections. Clearly, because our analysis focuses on several key, foundational aspects of IGSM in both theoretical and practical contexts, our conclusions are neither “generalizations” nor “extrapolations.”

7. “It is noteworthy that the California Department of Water Resources (DWR) has undertaken an independent review process, which includes theoretical documentation, upgrade, and enhancement of the IGSM for future uses in projects. As part of this process, a beta version of IGSM2 is released by DWR on September 6, 2002. This version includes daily time step for groundwater flow simulation and other refined features. The detailed documentation and verification of IGSM2 is expected to be released to the public in December, 2002.”

We concur with these reviewers in this regard. Perhaps the most noteworthy aspect of the work at DWR with regards to the conclusions in this report is that DWR has had to reformulate and rewrite most of the core of the IGSM solution methodology, as this report, and the work of Dr. LaBolle presented to these reviewers in the summer of 2000, suggested would be necessary.

C.2.2.2 Response to “3.0 Assumptions about IGSM”

This section of the reviewers comments describes “three important assumptions about IGSM” supposedly made in this report.

8. “3.1 The Report assumed that the stream aquifer interaction in IGSM is calculated on a monthly basis. This is not a correct assumption, as the IGSM uses a daily sub-step within a monthly time step to approximate the non-linear behavior of stream aquifer interaction. …”

The response to this comment was presented in the response to Comment #4, above. The reviewers are mistaken about what is stated in this report.
9. “3.2 The Report implies that the IGSM uses non-standard solution technique to solve the system of equations and therefore IGSM is error-prone. This assumption is also incorrect and the conclusion so derived is presumptuous. The IGSM uses a highly efficient numerical solution technique (See Method of Fractional Steps, by N. N. Yanenko, Springer-Verlag, New York, 1971) to solve the weakly nonlinear system of groundwater flow equations, obtained through Galerkin finite element formulation of the flow domain. This non-iterative method is one of the numerous solution techniques (iterative and non-iterative) used in solving groundwater flow problems (See Numerical Solutions of Partial Differential Equations in Science and Engineering, by L. Lapidus and G. Pinder, John Wiley and Sons, New York, 1982). The fractional step method has gained wide acceptance among the engineers and fluid dynamicists for its computational efficiency, convergence properties, and stability (See A Fractional Step Method for Unsteady Incompressible Flows on Unstructured Meshes, by G. K. Despotis & S. Tsangaris, Journal of Computational Fluid Dynamics, 1997).”

Unfortunately, Drs. Taghavi and Najmus of WRIME appear to have misinterpreted the analysis and conclusions of this report. Nowhere in this report is it stated that IGSM “uses non-standard solution techniques to solve the system of equations.” In fact, we concur with these reviewers that the solution technique used to solve the system of equations is of standard form and appears to be sound, although we are not aware of formal verification of any algorithm in IGSM. The overall solution methodology, however, involves more than the solution to a system of equations “obtained through Galerkin finite element formulation.” It involves more than 17,000 lines of seemingly unverified code. Indeed, it is the overall solution methodology of IGSM that, as results to the test problems show, is “error prone.”

The IGSM solution technique described in this report is a “method of fractional steps.” Its implementation in IGSM is not typical of the techniques of standard groundwater models. Results clearly show that when combined with the monthly time step in IGSM, the IGSM solution algorithm is not a virtue of the code, but rather, a real source of error in its solutions.

10. “3.3 The Report assumes that IGSM and MODFLOW code implementations and results are comparable on a one to one basis as long as the theoretical equations are the same, especially in the case of drain flow and aquifer stream interactions. This assumption is incorrect because IGSM and MODFLOW are two different models with two different numerical schemes. …”
We agree that IGSM and MODFLOW codes are different. This report acknowledges many differences between IGSM and MODFLOW in both section 2.1 Historical Background and section 3.0 Review of Theory. Nevertheless, our review of the governing equations that IGSM is supposed to solve show that they are comparable to those of MODFLOW with regards to drain and stream boundaries. Stream and drain boundaries are implemented as head-dependent boundaries within both MODFLOW and IGSM. This fact is key since the flow equation with head-dependent boundaries has unique solutions under the conditions considered herein. Further, MODFLOW is known to converge to such solutions. The problematic implementation of head-dependent boundary conditions in IGSM is detailed in this report in Section 3.2.3 Implementing Head-Dependent Boundary Conditions and partially explains the poor performance of IGSM in solutions to the test problems. Additional errors in solutions arise from implementing the “method of fractional steps” (see the response to comment #9 above) with a monthly time step, and other undocumented additions to the code (see Appendix B of this report for an example).

C.2.2.3 Response to “4.0 Formulation and Description of Test Problems”

In this section, Drs. Taghavi and Najmus of WRIME have three comments that focus on presentation of the test problems. The comments suggest that the IGSM model components tested herein and the applicability of test problems to “real world situations” are not described in this report. Yet the theoretical aspects of the code being tested are discussed throughout this report and the report states that “aquifer parameters are within a typical range.”

11. “4.1 The Report should include a discussion of the selection of these test problems including brief explanation of (a) why they have been chosen; (b) what model components are being tested; (c) what are the limitations of these test problems; (d) how are these test problems applicable or correlated with the real world situations encountered in the field. This will enable the reader to understand the context, scope, and limitations of the review.”

Our response to points (a) – (d) follows:

(a) Test problems were chosen based on findings from our review of the code. We have modified Section 1.2 and Section 4 of the report to more clearly convey this by adding “Example problems described herein are simple and represent common hydrologic phenomena. Problems were
chosen based upon a review of the code to demonstrate some of the known issues that would need to be resolved prior to conducting a more comprehensive verification effort.”

(b) The model components being tested are described throughout this report, namely the solution methodology, general head boundaries, drain boundaries and aquifer-stream interaction.

(c) The report states that test problems “do not constitute a complete analysis of the entire code.” A complete analysis would be a more extensive effort. Based on our knowledge of how the code was conceived and modified, we believe it likely that a more extensive review would reveal still more problems and issues with IGSM.

(d) Parameters of these test problems are comparable to those used in IGSM applications to “real world situations.” See the response to comment #1.

12. “4.2 It would be more helpful to first describe the test problems with adequate technical detail and figures before results are presented in Section 4 of the Report. This will be consistent with the CWEMF model review on 1-D Hydrodynamic and Transport Models (Sobey, 2001). In addition, it will enable readers to understand the relationships (or any lack thereof) between the conclusions and test problems.”

For each test problem, this report includes the governing boundary value problem, a figure depicting the model domain and boundary conditions, a table showing parameter values used, and a verbal description of the problem. Others have found this level of detail adequate. For example, staff at the California Department of Water Resources have been able to understand and verify all problems and results presented herein.

13. “4.3 The Report needs to explain why a triangular element mesh was chosen for IGSM instead of the simple rectangular grid similar to MODFLOW. IGSM is a finite element model capable of accommodating a rectangular grid. It seems that a similar grid in both models would provide a more equivalent comparison basis and remove another undesirable and unnecessary difference between the two models.”

It has been well-established in the literature that triangular finite elements applied with a rectangular grid produce conductance terms that are identical to block-centered finite difference
schemes like that of MODFLOW. The same cannot be said for quadrilateral finite elements; and thus triangular elements are a better choice for the IGSM runs in the example problems.

In Section 4. Example Problems, this report states that “Correspondence between nodal locations of the IGSM finite element mesh and MODFLOW finite difference grid used in all example problems is shown in Figure 4.1. The grids and boundary conditions are applied such that solutions computed at MODFLOW and IGSM nodes are comparable.” The triangular mesh was chosen to yield comparable solutions, as theory predicts. It is not within the scope of this report to detail the theoretical underpinnings of all of its content.

14. “It needs to be clarified why all MODFLOW simulations are performed with 25 time steps per month while comparing the results with those of IGSM, which simulates groundwater flow with 1 time step per month.”

The fact that IGSM is limited to one-month time steps is one of the many problems with IGSM discussed at length in this report. The choice of 25 time steps per month was a decision made on the part of the modeler that formulated the test problems. Note that time steps lengths are not equal, but start out small and increase throughout each month. The goal was to arrive at an accurate solution to the problem as a benchmark for IGSM. Variable time stepping, with increasing time step magnitude as the simulation progresses, is the well-established, standard approach. This procedure, which is standard in most groundwater flow modeling codes, arises from sound theory and from experience gained through practical applications.

C.2.2.4 Response to “5.0 IGSM Model Set-up for Test Problems”

15. “5.1 The IGSM data sets provided by the reviewers do not run with IGSM Version 5.0 as mentioned in the Report as the reference code. This incompatibility is not due to a simple code changes (such as a “WRITE” statement added to BOUND.FOR for printing drain outflow); rather, due to other changes that were made to IGSM for unspecified reasons. In reality, no code changes were necessary in IGSM Version 5.0 to run the example test problems; all it required is a comprehensive understanding of how IGSM data sets work. We recommend that all IGSM test problems be made compatible and rerun with IGSM Version 5.0 to remain consistent with the official release version of IGSM.”

These reviewers are not correct in that “no code changes were necessary in IGSM Version 5.0.” Referring to the response to comment #3, implementing the option of KOPTST = 1 in IGSM
requires recompiling the code. Before recompiling the code, non-standard compiler-specific FORTRAN coding had to be upgraded to standard code. These changes do not affect results of the code, but do change output file formats such that IGSM Pass 1 output files cannot be read by the original IGSM Pass 2, but only the code modified to open files in a standard way.

16. “5.2 The MODFLOW code was also changed to simulate the Example Problem Set 3: Stream Aquifer Interactions. This is a deviation from the original premise of the Review that the IGSM would be compared against standard MODFLOW code. “

The benchmark solutions from MODFLOW were used as a comparison because the code is known to converge to the specified boundary value problems. The minor modifications to MODFLOW were made to ensure that both codes were solving exactly the same problem. This does not constitute a “deviation from the original premise of the review”. Rather, it is a minor modification needed to make the two codes entirely comparable. See the detailed response to comment #3.

17. “5.3 The code changes made as part of the review process should be acknowledged and all changes to IGSM and MODFLOW codes should be provided in the Report (e.g. as an Appendix).”

This report “acknowledged” changes to MODFLOW in section 4.3.2 Simulation, stating “A modified MODFLOW stream package developed to incorporate such rating curves is used to compare solutions of MODFLOW with those of IGSM.” Actual changes and a manual are provided on the companion CD. Changes to IGSM are acknowledged in section 4. Example Problems: “All simulations are performed with KOPTST = 1, such that gradient calculations in the computation of seepage losses from streams are consistent with those of MODFLOW and the theory discussed herein.” Other minor code changes to IGSM to remove non-standard file OPEN statements are detailed on the companion CD and do not affect results. It is doubtful that most readers have an interest or the training to appreciate these minor code changes. It was felt that the CD was best place for such information. Nevertheless, this Appendix also records these changes.
18. “5.4 The use of KOPTST=1 flag in all IGSM simulations as mentioned in Section 4 of the Report is erroneous because KOPTST=1 flag in IGSM is reserved for unsaturated stream seepage. The physical set-up of the example problem set no. 3 indicates saturated stream seepage conditions, which is simulated in IGSM by setting KOPTST=0. This error should be corrected.”

A review of the IGSM and MODFLOW codes shows that the IGSM option of KOPTST = 1 is consistent with the theoretical foundations of MODFLOW. The footnote of Section 3.2 Solution Methodology states “IGSM contains a parameter KOPTST within the code that controls how head gradient terms in the stream aquifer interaction are computed. All simulations are performed with KOPTST = 1, which yields (like MODFLOW) a gradient calculation based on atmospheric pressure (unsaturated conditions) at the bottom of the stream bed when the average hydraulic head is below the elevation of the bottom of the streambed.” Use of the option of KOPTST = 1 in IGSM does not require unsaturated conditions beneath the stream channel, but treats conditions as if they are unsaturated should the head of the connecting aquifer node fall below the elevation of the bottom of the streambed. Use of KOPTST = 1 is not an error in the analysis. Again, CA DWR is using these test problems with success.

19. “5.5 The addition of the “WRITE” statement to BOUND.FOR to output the drain flow was done incorrectly due to the printing the wrong variable. This error should be corrected.”

Our review and an independent review by staff at DWR find that this write statement is indeed correct and correctly outputs the flux that enters into the right-hand-side vector as a result of applying the drain boundary condition.

20. “5.6 The initial conditions of problem sets No. 3 is set to 200 ft in the entire domain, while the MODFLOW solutions show an instantaneous steady state condition along the stream with an elevation of about 184 feet. This is an inappropriate initial condition.”

These reviewers are mistaken: Initial conditions in MODFLOW simulations of test problem 3 are 200 ft. One month is sufficient for heads to equilibrate to a near steady-state condition in this example problem, which explains these reviewers’ observations and misinterpretations.
C.2.2.5 Response to “6.0 Interpretation of IGSM and MODFLOW Results”

In this section, Drs. Taghavi and Najmus of WRIME comment on the “… interpretation of the results of selected test problems …” These reviewers state that “In order to understand the significance of the findings presented in the Report, we conducted additional model runs of selected test problems. The results of these model runs are used to interpret and explain the differences between IGSM and MODFLOW, as summarized below.” The authors of this report were provided with the code used to make these additional runs. A review of this code shows that it has been modified to bypass problems identified in our report. Reviewers also incorrectly plot results from this modified code, making IGSM results appear in phase with MODFLOW results, when in fact they are not. Detailed responses are given below.

21. “6.1 The problem set-up of a general head boundary condition at a 1-ft distance from both the model boundaries (Problem Set No. 1-b) is not an appropriate set-up for testing implementation of general head boundary conditions, because it is equivalent to a specified head boundary due to immediate proximity to the boundaries ... The results from a more representative general head boundary (e.g. a stream) about 1 mile (5,000 feet) away from boundary are shown in Figure below. A comparison between the results shown in this Figure and that shown in Figure 4.1.4 in the Report indicates that a well-posed problem provides reasonable simulation results.”

General head boundaries (GHBs) are common to many groundwater flow simulations and implemented with a wide range of parameters, including those that more closely resemble a specified head as was implemented in problem set 1. A working code should be able to simulate GHBs over an entire range of parameters. The reviewers have presented a simulation with the head boundary located 5000 ft from the boundary domain. Examination of these reviewers’ plot of IGSM results for this problem shows that IGSM solutions fail to come close to the accurate MODFLOW solution. Further, we are unable to reproduce reviewers’ results. A correct plot of the simulation results with a GHB distance at 5000 feet is provided below. As one can see, results from these reviewers’ example problem do not suggest that IGSM produces “reasonable simulation results.”
Furthermore, the reviewers have overlooked a fundamental of general-head boundary conditions. The 1-ft distance referred to is but one term in a groundwater model’s computation of conductance, which is the key term that connects the fixed head (representing, for example, head of a stream, local head, or head in an overlying aquifer) to an adjacent aquifer node in the model. Conductance is computed as the product of hydraulic conductivity and cross-sectional area of flow, divided by length (distance) of the connection. In reality, this term can vary over many orders of magnitude from case to case due to natural variations in hydraulic conductivity alone. Thus, a distance of 1 ft in Problem 1-c and 1-d results in a conductance term that is plausible for either a specified head condition or for many other hydrogeologic conditions, such as stream-aquifer connection in which the streambed is composed of higher permeability material than the aquifer (note that the reviewers have incorrectly referred to problem 1-c as problem 1-b). Such a conductance might also arise through the use of the general-head condition for modeling vertical leakage across a thin aquitard. Clearly, problems 1-c and 1-d are well-posed.
22. “6.2 We were unable to replicate the results presented in Figure 4.2.5. It seems that there are unit conversion errors in that Figure; there are also labeling errors in that Figure.”

Figure 4.2.5 has changed from the draft sent to these reviewers due to the unit conversion error called to our attention by Can Dogrul of DWR. These reviewers were alerted to this oversight prior to developing their comments and this final report does not contain the error noted.

23. “6.3 After erroneous “WRITE” statement in BOUND.FOR is corrected to output the appropriate variable for drain flows and results are interpreted correctly, the test problem (2c) results look like what is shown in Figures below. A comparison of the results shown in the Figure above with that in the Figure 4.2.4 of the Report shows that the latter is due to printing drain flows from the wrong variable in the IGSM code.”

Our review of, and review by Can Dogrul of CA DWR of, the WRITE statement in question finds that it is not in error, but instead correctly outputs the drain-boundary flux that enters the right-hand-side vector. The plots contained in this report for drainflow results from IGSM are correct, and reproducible by CA DWR. As explained in section 4.2 and Appendix B of this report, IGSM 5.0 computed drainflows do not necessarily depend on specified drain boundary conductance. This is a very serious theoretical and coding error.

As explained in the introduction, the task assigned to reviewers was to assess the work in this report developed with IGSM 5.0. However, Drs. Taghavi and Najmus in developing their comments used a version of IGSM modified to bypass the problem with the code identified in Appendix B of this report. As their comment above shows, these reviewers fail to inform the reader that this modified code is being used their analysis. Reviewers suggest that results in this report are wrong because they do not compare with results from this modified code. The specific code change in question is compared with the original IGSM 5.0 (the BOUND.FOR code provided by these reviewers) below:
IGSM 5.0

IF(CB.GE.0.) THEN
  C=AMIN1(C,ELG(ID-LD))
  C=AMIN1(CB,(C-HN(ID))*AS(ID)-QD(ID))
  C=AMAX1(0.,C)
ELSE
  C=AMAX1(C,DH(ID))
  C=AMAX1(CB,(C-HN(ID))*AS(ID)-QD(ID))
  C=AMIN1(0.,C)
ENDIF

CODE MODIFIED TO BYPASS A PROBLEM IDENTIFIED IN THIS REPORT

IF(CB.GE.0.) THEN
  C=AMIN1(C,ELG(ID-LD))
  C=AMIN1(CB,(C-HN(ID))*AS(ID)-QD(ID))
  C=AMAX1(0.,C)
ELSE
  C=AMAX1(C,DH(ID))
  C=AMAX1(CB,(C-HN(ID))*AS(ID)-QD(ID))
  C=AMIN1(0.,C)
ENDIF

C FOR DRAINFLOW, ADJUSTMENT FOR EFFECTIVE FLOW AREA SHOULD NOT BE MADE
IF(KMBT(I) .LE. -10000) C=CB
ENDIF

The variable C is the boundary flux term. The variable CB is also a flux term defined previously in the code. The additional line of code shown in bold tells the code that if this boundary is a drain boundary, then make the variable C equal to the variable CB. This change eliminates changes to C made in the code and replaces it with the value in CB. This modification bypasses the problem with the code identified in Appendix B of this report. Yet serious problems still remain that compromise the ability of IGSM to simulate drainflows.

In addition to using a code modified to eliminate a problem identified in this report, these reviewers have incorrectly plotted IGSM drainflow results from this modified code, again making the results appear better than they actually are. The total drainflow in the course of a month should be plotted for that corresponding month. These reviewers have plotted IGSM drainflow results shifted by one month, making IGSM results for oscillating drainflows, that are out of phase with the correct MODFLOW solution, appear in phase. One can clearly identify this error because IGSM monthly drainflows are plotted by the reviewers for month 0, when no time has elapsed. Shift the IGSM results by one month and one obtains a correct plot of IGSM drainflows from the code modified to improve results. The plot below summarizes the findings herein. As one can see, the results of the modified code still compare poorly with those of MODFLOW as drainflows are out of phase, resulting in errors as large as 50% in month 10, for
example. Importantly, as explained in section 4.2 and Appendix B, drainflows of the unmodified IGSM 5.0 do not depend on conductance, as they should.

![Graph showing MODFLOW, IGSM 5.0, and modified IGSM code used by reviewers flow rates over months.](image)

24. “6.4 In problem 3.1a, a reduction of flow from 500 cfs to 50 cfs removes the oscillations from the solution as shown in Figure below. Similarly, an increase of Dx from 1000 ft to 5000 ft without changing the inflow of 500 cfs also provides a stable solution as shown in Figure below.”

We concur with these reviewers, oscillations can be reduced by changing inflows and discretization. Lack of oscillations does not imply an error-free or accurate solution. As one can see from the reviewers’ plot of groundwater head, IGSM results still compare poorly with the correct MODFLOW solution. In addition, the user rarely has the luxury of choosing to reduce streamflow to eliminate oscillations in a solution and the suggestion that this be used as a remedy is nonsensical. Finally, Dr. LaBolle’s original work presented to these reviewers in the year 2000, which has since been published (LaBolle and Fogg, 2001; see references of this report) and undergone two independent peer review processes, shed light on the many problems with IGSM and included a demonstration that significant errors and oscillations can occur with large grid spacings (e.g., 10,000 ft) for typical aquifer and stream parameter values.
25. “6.5 Report should provide a comparative discussion of the differences in formulation of stream aquifer interaction between IGSM and MODFLOW and explain the differences in results between two models. It should be noted that MODFLOW solves the system of equations simultaneously as a fully coupled system; as a result, it cannot constrain the seepage loss to a limit that may be governed by the maximum conveyance capacity of the streambed material. This limitation of not being able to impose a physical constraint due to the numerical scheme of MODFLOW is discussed in the MODFLOW documentation and the users are cautioned about choosing the correct conductance parameter value (Chapter 6 of MODFLOW Documentation published by USGS). On the other hand, IGSM takes a different approach and actually puts relevant physical constraints a priori and implements a locally iterative daily sub-stepping scheme to account for the nonlinear nature of the stream aquifer boundary. The differences in numerical schemes should be clearly presented before any comparison of two models is attempted and accepting one or the other as correct.”

Firstly, section 3.0 Theoretical Review of this report compares, quoting from these reviewers, “differences in formulation of stream aquifer interaction between IGSM and MODFLOW.”

Secondly, these reviewers have mischaracterized the MODFLOW solution technique. MODFLOW does not solve the system of equations from the stream and aquifer interaction “simultaneously as a fully coupled system.” Instead, MODFLOW implements stream and aquifer modules sequentially within a time step. Importantly, however, MODFLOW iterates within a time step to converge the solution. IGSM lacks this iteration. This iterative technique is referred to as quasilinearization, as explained in this report. It is different than solving the system simultaneously as a fully coupled system.

Thirdly, the MODFLOW stream boundary is physically based, with flux controlled by conductance and head gradient. IGSM attempts to solves the same equations, as explained in section 3.0 Theoretical Review of this report. Moreover, MODFLOW’s stream-aquifer interaction module does indeed limit the streambed seepage based on the conveyance capacity of streambed material.

Finally, Figure 4.3.4a is presented below to remind the reader how poorly IGSM performs to these test problems. The suggestion that IGSM may be correct in this case, and MODFLOW simply different, is completely baseless.
Figure 4.3.4a. IGSM and MODFLOW solutions for streamflow plotted against time for problem 3c, $C_R/P_wL_R = 100.0$ day$^{-1}$, scenario 1 steady stream inflow of 500 cfs, no pumpage. IGSM solutions deviate significantly from those of MODFLOW.

26. “6.6 The Report needs to explain why MODFLOW results show an instantaneous steady state condition …”

In fact, MODFLOW results are not at an instantaneous steady state condition; rather, heads and flows equilibrate during the first month, but results are only plotted monthly making results appear steady (see the fig. above for example).

C.2.2.6 Response to “7. Validity of Conclusions of the Report”

Here, reviewers suggest that “test problems … have no connection to a real world situation” when in fact, as our response to comment #1 shows, model parameters are within the range of those used in IGSM applications and commonly found throughout California. Further, the hydrologic phenomena modeled are common to most IGSM applications. Detailed responses are given below.

27. “7.1 The Report concludes that the “IGSM results are commonly error plagued”, without qualifying which results (all? or some? or those tested?). In addition, the
Report does not inform the reader about the broad base of test cases, which led to the use of the word ‘commonly’.

This draft report reviewed by WRIME states the following: “The findings of this report strongly suggest that IGSM model results are commonly error plagued.” This statement has been replaced with the following for clarification: “The findings of this report show that several key algorithms used in IGSM differ substantially from the standard, tested methods employed by mainstream groundwater modeling codes. Furthermore, we demonstrate that, as a direct consequence of these non-standard features, IGSM model results can contain significant errors for typically encountered hydrologic conditions, both at the local and regional scales.” The response to comment #1 (see above) shows that conditions and model parameters used in this analysis are within the range of those in IGSM applications, and the hydrologic phenomena modeled occur in all IGSM applications with which we are familiar. Therefore, as the statement strongly suggests, it is all IGSM applications to real systems that are problematic. The degree of error will vary, of course. However, it is not within the scope of this report to assess errors in actual IGSM applications due to the problems identified, and remaining problems not identified, with IGSM.

28. “7.2 The Report needs to explain why results of the simple examples that hardly represent field conditions are also applicable to the actual field scale applications. Can test problems conclusions be extrapolated to field scale applications? What types of similarities do the test problems bear with the real world applications of IGSM? Can the unrealistic conditions used in test problems really occur in real world or has occurred in any of the previous IGSM applications?”

The response to comment #1 (see above) shows that conditions and model parameters used in this analysis are within the range of those in IGSM applications, and one or more of the hydrologic phenomena modeled occur in each IGSM applications with which we are familiar. See also the response to comment #29 below.

29. “7.3 The Report needs to address whether any reviews were conducted on actual field applications of IGSM before reaching generalized conclusions about IGSM.”

As our response to comment #1 shows, parameter ranges in this report are comparable to those used in IGSM field-scale models. Therefore, the results of this review apply to such model applications and this report does not need to “show” that reviews of field-scale applications were
performed before reaching conclusions herein. Nevertheless, this response will serve to inform readers that such a review has been conducted. To assess errors in an IGSM field scale application, one would have to build a comparable model with code known to converge. Dr. LaBolle made such an assessment in the year 1999 with the Central Valley Groundwater and Surface Water Model (CVGSM). In this comparison, a comparable model was built with the code FEMFLOW3D. Sources and sinks to groundwater and streamflows in the CVGSM were extracted and used as inputs and outputs to FEMFLOW3D. Results showed dramatic differences in aquifer stream interaction, consistent with the findings of this report.

30. “7.4 In Section 5.7 of the Report, the reviewers state that “findings of this report strongly suggest that IGSM model results are commonly beset with errors”. In our view, this is an extrapolative and over generalized comment based on very simple example test problems that have no connection to a real world situation. The use of words/phrases such as “commonly beset with errors” requires more extensive testing with a variety of problems.”

See responses to comments #28 and #29.

31. “7.5 In Section 5.7.1 of the Report, the reviewers bring up the issue of the number of lines of the IGSM code when discussing modeling alternatives. It is not clear why length of the source code of a model is even a concern or an evaluation criterion for the upgradability of the IGSM.”

The length of code has significance when one considers the time and effort necessary to document, understand, and test such code. We are quite certain that staff at the CA DWR will attest to the difficulty of this job as they undertook such an effort during the course of this review. Apparently, as this review has found, such an effort has been lacking with IGSM. In this sense, IGSM is an unfinished model development effort, unfit for public dissemination.

32. “7.6 In Section 5.7.1 of the Report, the reviewers state that “. Conclusions of this report point to several drawbacks with fixing IGSM”. However, the drawbacks to fixing IGSM are not stated anywhere in the Report. These drawbacks and how they weigh in with other recommendations need to be clearly stated for the sake of completeness of the Report.”

Section 5.7 has been edited for greater clarity.
33. “7.7 In Section 5.7.3 of the Report, the reviewers state that the problems considered by IGSM can be solved by alternative modeling platforms that are known to be reliable, but does not give names of any such platforms. In support of the above statement, the Report should provide the name(s) of the specific code that (i) can simulate the suite of hydrologic processes included in IGSM and (ii) has been successfully applied. This will make the statement more than an opinion or a passing comment and bring completeness to the Report.”

For clarity, we have expanded section 5.7.3 to include a table comparing IGSM with two other codes. IGSM is only used to solve a small fraction of the groundwater flow problems being solved today. Our report states that “The problems currently considered by IGSM can be solved with alternative modeling platforms that are known to be reliable.” Problems solved with other codes commonly include settings with rainfall, runoff, crops, and unsaturated zones. Those interested in how groundwater modeling is now performed with other applications can refer to the numerous reports (e.g., reports by USGS, numerous consulting firms, and state geological surveys) detailing groundwater models and their development.

Convenience of data entry and purported functionality are not reason enough to select a model platform. On the contrary, knowledge that a model platform is unreliable is reason enough to reject it. Therefore, the fact that IGSM contains modules for various hydrologic processes is of little utility because we have shown IGSM to be fundamentally unreliable.
C.3 Response to Comments of Rich Juricich, CA DWR

1. “The authors seem to dismiss both the regional scale of the studies to which the IGSM model is applied, and the level of accuracy that is generally available for such studies.”

The comment implies that when the data contain significant errors, having a fairly error-free numerical solution is less important. There is some merit to this notion when the errors in the numerical solution are small relative to errors in the data. As shown in examples, however, the numerical errors in IGSM can range from moderate to quite large, regardless of the scale of the model. What distinguishes a regional scale model from more local scale models is the node spacing. In the report we used both intermediate- (100 ft) and regional-scale (1,000 ft) node spacings specifically to examine significance of the errors at different scales. Dr. LaBolle presented IGSM simulations in the Sacramento office of the California Department of Water Resources, in the summer of 2000, with 10,000 ft node spacing that show similar errors. Furthermore, the existence of inaccurate and uncertain data does not warrant unnecessarily introducing further inaccuracies in calculations and uncertainties in model solutions due to choice of solution technique. On the contrary, if significant inaccuracy and uncertainty enters a model solution from both the data and the solution technique, one is left with the very difficult, if not impossible, task of accounting for each in model solutions. In other words, dealing with uncertainty and inaccuracy in the data is challenging enough without unnecessarily introducing still additional errors through the solution technique.

When faced with modeling complex systems with uncertain data, groundwater modelers have always been able to, at the very least, rely on numerical accuracy of their flow models. This, in turn, allowed hydrologists to use the flow models for very productive hypothesis testing even when the data were not sufficient to construct a predictive model. In fact, it has been shown repeatedly that the greatest strength of groundwater models is their hypothesis-testing and conceptual modeling capabilities, not their predictive capabilities. These strengths disappear if errors in the solution technique are significant, thereby undermining the primary benefit of groundwater flow modeling and reversing years of progress wrought through development of modeling code and of modeling ethics.
2. “The authors have not demonstrated that the linearization technique used by IGSM is unreasonable given the regional scale of the IGSM applications. Since the authors have not compared a regional IGSM application to a regional MODFLOW application they cannot fairly comment on the ability or inability of IGSM to evaluate the kinds of problems for which it is used.”

On the contrary, we have shown that IGSM lacks numerical methods that reliably solve the governing equations (characterized by coupled models, e.g., aquifer and stream, or land use, aquifer and stream) and ensure convergence and mass balance. These aspects of the problem are common to all groundwater flow models, regardless of scale. Again, we have implemented the examples with node spacings typical of regional-scale models. Earlier in our study we had voiced the desire to do a MODFLOW-IGSM comparison on the Sacramento County model, but the time and resources needed to do that were beyond the scope of this project. Furthermore, it became clear to us during our tests of IGSM that we could produce regional-scale phenomena in our simple examples by manipulating the node spacing, as demonstrated in the report.

3. “The standard calibration practice is to constrain the parameters being calibrated to a range that is reasonable based on known data. If a particular application of IGSM follows this practice and matches the historical data reasonably well then it is reasonable to apply the model to studies within the bounds for which it was designed.”

It is fallacious to assume that a model’s ability to match historical data alone establishes a well-calibrated model because a well-calibrated model is a subset of models that agree well with observations. Experience as well as exhaustive studies repeatedly demonstrate that seemingly calibrated groundwater models can drastically misrepresent actual groundwater conditions (see Konikow, L.F., and J.D. Bredehoeft, Ground-water models cannot be validated, Advances in Water Resources, 15: 75-83, 1992). For example, in most “calibrated” groundwater flow models, the simulated and measured water levels can match quite closely, even if recharge or discharge rates are off by orders of magnitude. This sort of thing can arise from errors in model parameters and the nonuniqueness problem that is inherent to model calibration.

Thus, it is untrue that “If a particular application of IGSM follows this (standard calibration) practice and matches the historical data reasonably well then it is reasonable to apply the model to studies within the bounds for which it was designed.”
This problem only gets worse if one attempts to calibrate a model that is introducing errors due to the numerical solution technique. The exercise of calibration assumes *a priori* that the numerical technique reliably solves the problem posed, i.e., the governing equations. If we add nonuniqueness to the other problems in IGSM, in which it is not clear that we are solving the problem posed by the user, or indeed perfectly clear that we are not solving the problem posed, then it appears that little can truly be gained from the exercise of calibration; “calibration,” in this case, will not yield a more reliable model result, but rather, only a solution that agrees better with observations. Such a model is not necessarily reliable or well calibrated.

4. “It appears that the problems set up for the UC Davis peer review violate the stability criterion that is specified in the paper by the authors.”

The reviewer appears to have missed an important point: IGSM is inherently inaccurate and often unstable under a variety of conditions that are difficult to ascertain or predict. The IGSM users, even after years of use, have never been warned of this problem in written or oral communication until now by our report, and in the year 2000 by Dr. LaBolle. The report discusses undocumented additions to the code that violate fundamental principles of mass balance and that were added to try to stabilize the error-prone algorithms of IGSM (see Appendix B for a detailed example). Further, it appears that parameters in IGSM models were sometimes adjusted to compensate for errors due to the instability, when instability, not parameter error, was at the root of the problem. This is most unfortunate.

In a code that functions properly, the criterion presented in our report is commonly used for choosing a combination of initial time step and node spacing that minimize accumulation of error in the solution. As the solution progresses in time during a stress period, time step can increase from this initial value while still maintaining accuracy. However, when stresses change significantly in time, it may be necessary to return to this accuracy criterion to determine time step. This accuracy criterion corresponds to the stability criterion commonly given for an explicit algorithm.
By presenting the accuracy/stability criterion we were not trying to provide a framework for properly choosing node spacings and time step sizes in IGSM that result in stable solutions (especially since the time step in IGSM is fixed and lack of stability and accuracy in IGSM solutions is a complex problem involving other problems with the code, including its inability to simultaneously converge coupled models). That would effectively reverse years of groundwater modeling progress, since there is no reason in this day and age to have a groundwater flow model solution that is anything less than unconditionally stable. In other words, stability should not be an issue with IGSM at all, but it is. We chose representative combinations of parameters, node spacing and boundary conditions for CA conditions to demonstrate this problem and to place it in perspective. We emphasize that no state-of-the art groundwater flow models today require that grid spacing or time-step size be manipulated to insure stability, because since the 1970’s it has become a scientific “given” that all these codes are unconditionally stable. The use of an unstable numerical scheme in IGSM is both backward and unnecessary. Finally, reducing the time-step size (which is not possible in IGSM) or increasing node spacing in IGSM only partially addresses one of the many problems found with the code, and the latter may introduce further inaccuracies in solutions as discussed in the report. For examples with larger 10,000 ft node spacing see LaBolle and Fogg (2001) referenced in this report.

5. “The authors suggest in a cursory manner that there are other modeling platforms available that will perform the same function as IGSM. Much more discussion needs to be given to this idea.”

For clarity, we have expanded section 5.7.3 to include a table comparing IGSM with MODFLOW 2000. We note that IGSM is only used to “solve” a small fraction of the groundwater flow problems being solved today. Problems solved with other codes commonly include settings with rainfall, runoff, crops, and unsaturated zones. Those interested in how groundwater modeling is now performed with other applications can refer to the numerous reports (e.g., reports by USGS, numerous consulting firms, and state geological surveys) detailing groundwater models and their development.

6. “IGSM includes a variety of modules not directly available in other groundwater models like MODFLOW. These include rainfall-runoff simulation, crop water use, soil moisture accounting, unsaturated zone simulation, and reservoir operations ... In addition, data requirements for these alternative models must also be discussed. It is likely that the data requirements are too onerous to practically model a regional
hydrologic system with all the functionality that IGSM has and to the level of sophistication desired by the authors.”

For clarity, we have expanded section 5.7.3 to include a table comparing IGSM with MODFLOW 2000. Convenience of data entry and purported functionality are not reason enough to select a model platform. On the contrary, knowledge that a model platform is unreliable is reason enough to reject it. Therefore, the fact that IGSM contains modules for “rainfall-runoff simulation, crop water use, soil moisture accounting, unsaturated zone simulation, and reservoir operations” is of little utility because we have shown IGSM to be fundamentally unreliable.

MODFLOW and other codes have been used for decades to simulate regional groundwater flow. Unsaturated flow simulation is available for many other codes, including MODFLOW. MODFLOW, for example, includes a LAKE module linked to the stream package that can be used to simulate reservoir operations. Importantly, all of these modules are tested, verified and widely supported. Surface budget accounting calculations (for rainfall, runoff and crop water use) are commonly performed external to a groundwater code. Nevertheless, it is quite feasible to create these simple surface budget modules for use with a tested and verified code.