Uncertainty in Hydrologic Modeling and the **Curse** of Dimensionality

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ACCOUNTING FOR UNCERTAINTY IN HYDROLOGIC MODELLING

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- UNCERTAINTY IN MODELLING

  Causes or Sources

  - natural randomness
  - estimation
  - scale of representation
  - omissions

- ACCOUNTING FOR UNCERTAINTY

  Approaches

  - Monte Carlo simulation
  - stochastic equations
  - integration
  - calculus of propagation
  - compensating tricks

PASSAGE from the "PUNCTUAL"

to the "PARCEL"

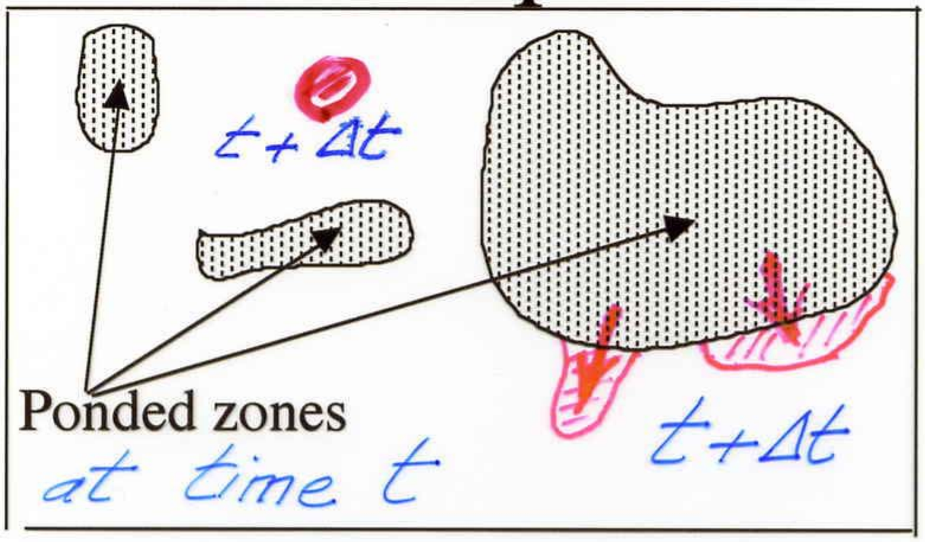
ENTAILS (ARBITRARY) CHOICE OF

- **PUNCTUAL, PLAUSIBLE, PHYSICAL LAW**
- **PLAUSIBLE LAW OF CHANCE**

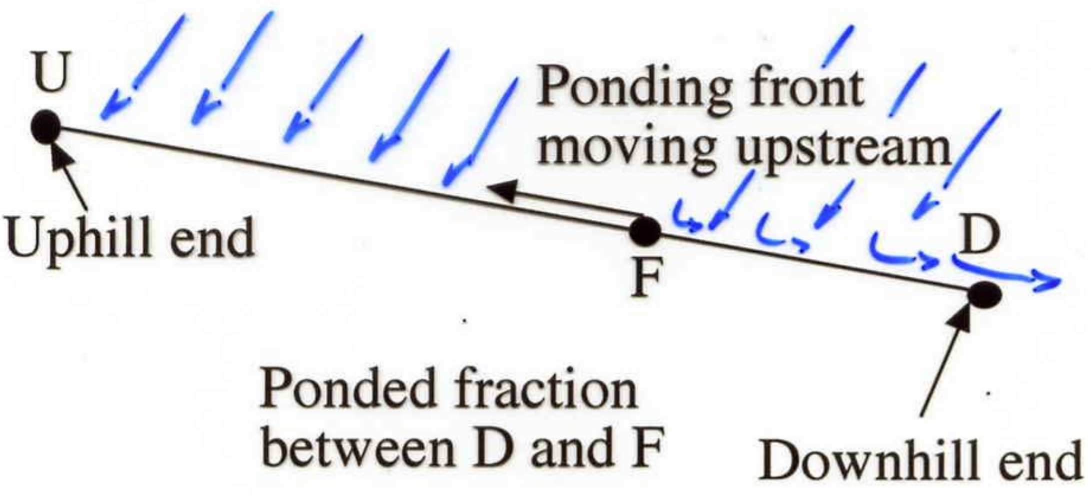
Given "**capacity** infiltration rate" and ponding time formulas at a **point scale**, and laws of **chance** for the distribution of conductivity, one can derive **analytically** the infiltration rate for a plot or a parcel.

In this derivation it is **necessary** to account for runoff from the ponded surfaces that will cause **"run-on"** and thus **opportunity** for infiltration in zones where all the rainfall had infiltrated but some **unfilled capacity** still exists.

Random pattern



Organized pattern (such as KDD)



Typical (possible) formulae

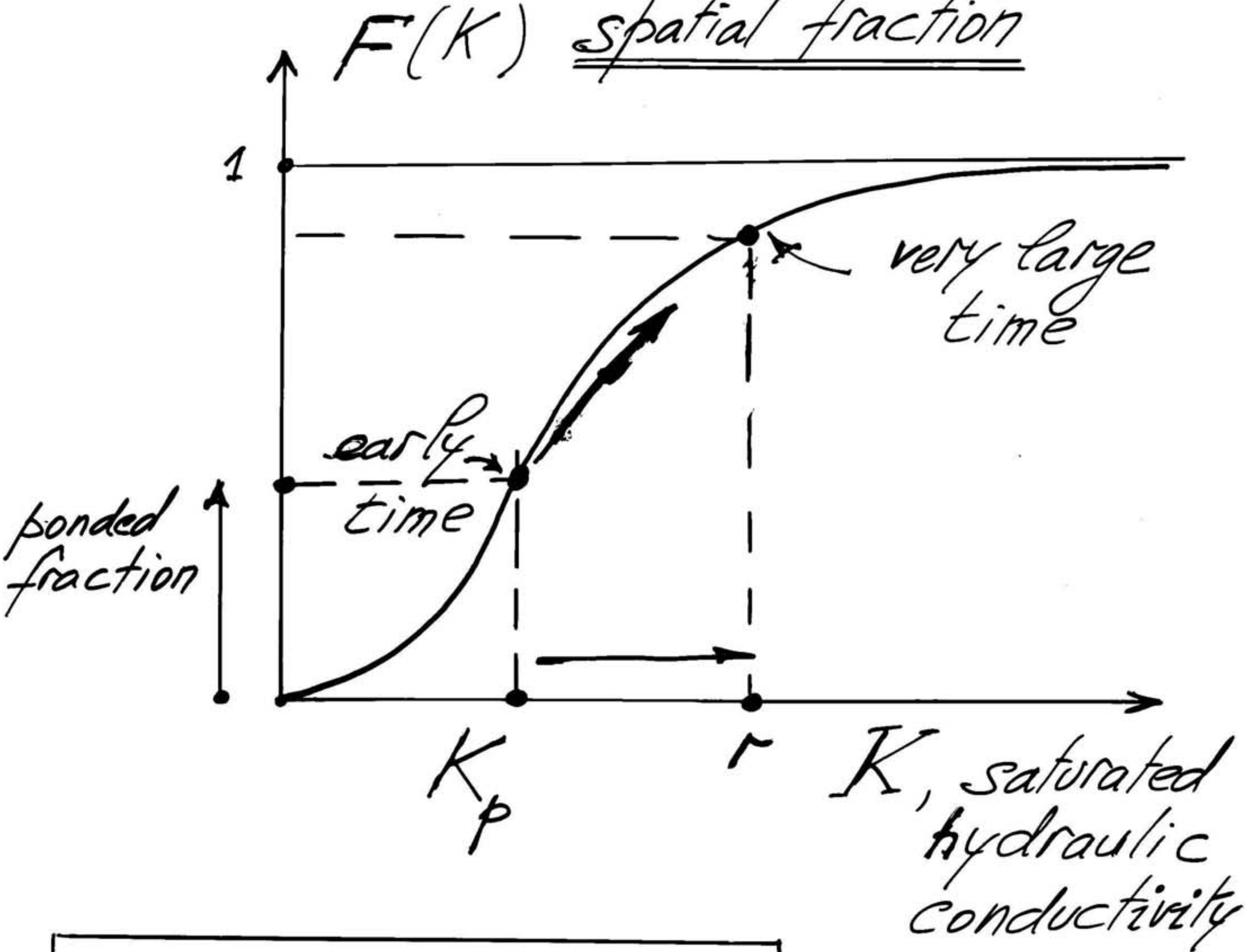
Ponding time : $$t_p = \frac{K S_f}{r(r-K)}$$

with storage-suction factor:

$$S_f = (\tilde{\theta} - \theta_i) \int_0^{h_{ci}} k_{rw} dh_c = (\tilde{\theta} - \theta_i) \underline{\underline{H_{cMi}}}$$

and capacity infiltration rate:

$$i_c = K + (r-K)e^{-k(t-t_p)}$$



$$K_p = \frac{r^2 t}{r t + S_f}$$

S_f : storage - suction factor

$$(\tilde{\theta} - \theta_i)\, H_c$$

$\tilde{\theta}$: natural saturation

θ_i: initial

H_c: effective capillary drive

Parcel infiltration rate

Over the ponded fraction

$$\bar{i}_{pfP} = \frac{\int_0^Z i_c(K,t) f(K) dK}{F[Z]}$$

For the entire parcel:

$$i_P = \{1 - F[Z]\} r + \int_0^Z i_c(K,t) f(K) dK$$

The **functional** form of infiltration for

parcel $\neq$ column

(SPATIAL) DISTRIBUTION OF CONDUCTIVITY

Density function for K: $f(K)$

cdf : $F(K)$.

"RULES OF THE GAME"

Scenarios as to the fate of the excess rain, i.e. the water that did not infiltrate.

Instantaneous Removal (by Microchannels) IRS

K Decrease Downhill KDD

K Increase Downhill KID

Single Huge Uniform Column SHUC

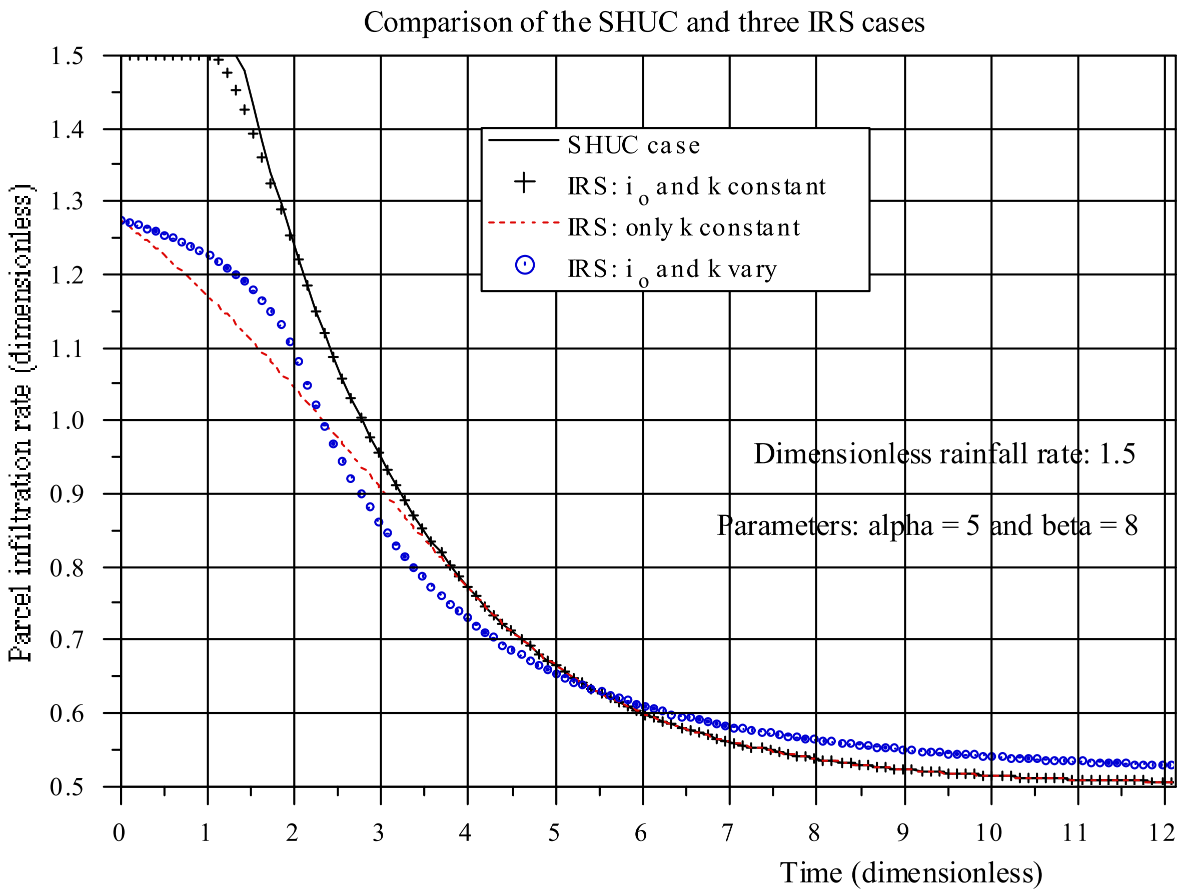


Figure 1. Infiltration rates for the SHUC case and three IRS cases

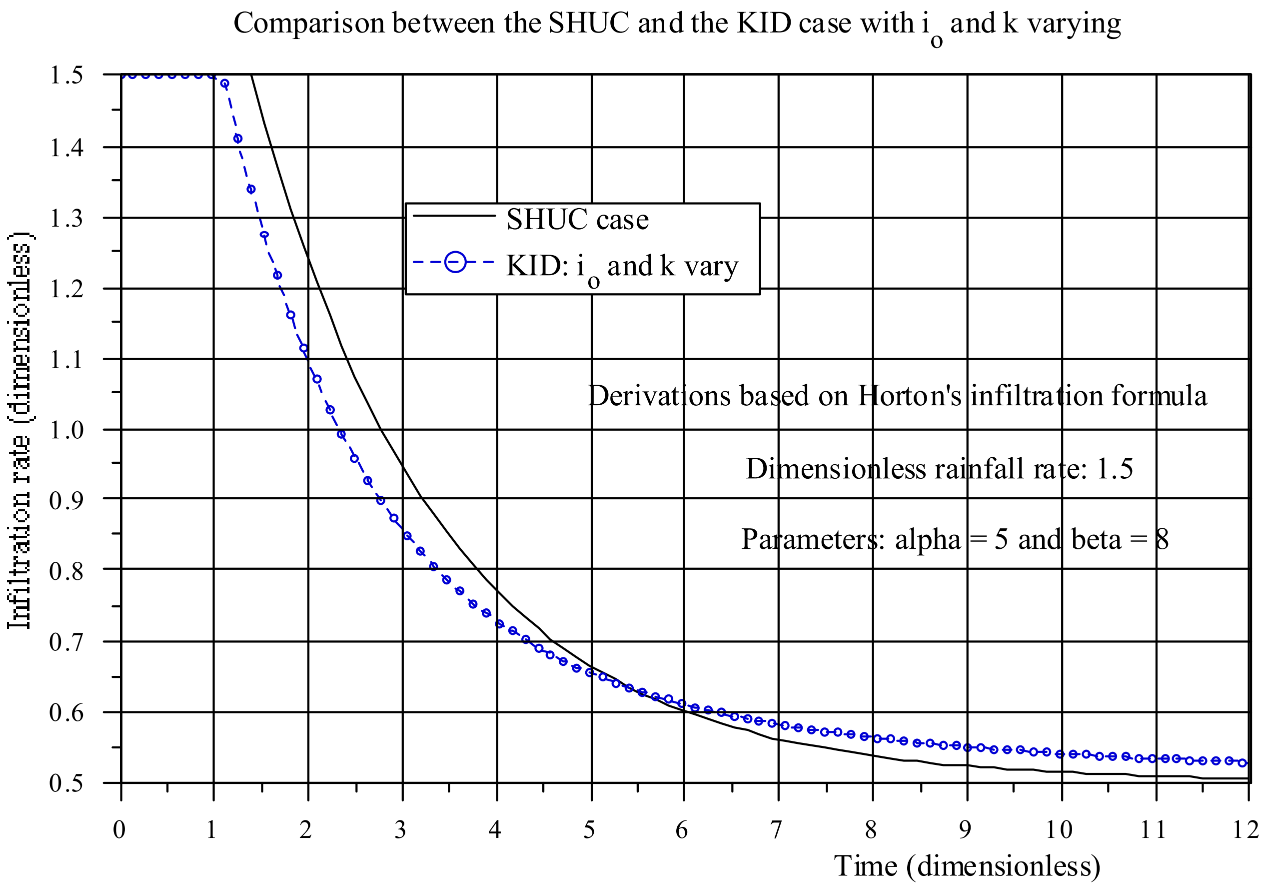


Figure 2. Infiltration rates for the SHUC case and the KID case when all parameters vary.

CONCLUSIONS

WHICH SCENARIO TO USE FOR PARCEL INFILTRATION? A MIX.

fraction f_μ of the parcel well drained by microchannels

$$i_P = f_\mu i_\mu + (1 - f_\mu) i_S$$

More precise derivations will **not** significantly improve the description of the phenomena as opposed to knowing **better** the **fraction drained by microchannels versus that subject to run-on.**

Stochastic (Groundwater) Equation. #1

Classical deterministic Boussinesq equation

$$S\frac{\partial H_r}{\partial t} - div.[T_r(grad.H_r)] = q$$

with usual symbols but with the added subscript r to indicate that such a subscripted variable is actually a **random** variable. To simplify let us only consider the **steady-state** case in just **one** dimension and with **no** recharge or withdrawal rate within the domain.

$$\frac{\partial}{\partial x}[T_r(\frac{\partial H_r}{\partial x})] = 0$$

Stochastic (Groundwater) Equation. #2

$$\frac{\partial}{\partial x}[T_r(\frac{\partial H_r}{\partial x})] = 0$$

Define: $H_r(x) = H(x) + h(x)$

H(x) is the **mean** value of H_r(x).

The expected value of h is zero.

Similarly: $T_r(x) = T_L(x)e^{\theta(x)}$

The expected value of θ is zero.

The **random** variable θ is defined by its variance: σ^2_θ(x) and covariance: $C_{\theta,\theta'}(x, x')$.

Given values and functions.

Stochastic (Groundwater) Equation. #3

$$\frac{\partial}{\partial x}[T_r(\frac{\partial H_r}{\partial x})] = 0$$

Substitution (with previous definitions) :

$$\frac{\partial}{\partial x}[T_L e^{\theta} \frac{\partial H}{\partial x} + T_L(\frac{\partial h e^{\theta'}}{\partial x})_{x'=x}] = 0$$

Taking expectation, defining: $T_A = T_L e^{\frac{1}{2}\sigma_\theta^2}$

$$\frac{\partial}{\partial x}[T_A \frac{\partial H}{\partial x} + T_A(\frac{\partial C_{h\theta'}}{\partial x})_{x'=x}] = 0$$

Problem? One equation and **two** unknowns:

$H(\mathbf{x})$ and $C_{h\theta'}(\mathbf{x},\mathbf{x}')$

Stochastic (Groundwater) Equation. #4

Premultiply the stochastic equation

$$\frac{\partial}{\partial x}[T_L e^{\theta}\frac{\partial H}{\partial x} + T_L(\frac{\partial h e^{\theta'}}{\partial x})_{x'=x}] = 0$$

by random variable $\theta'(x')$ and take expectation leading eventually to:

$$\frac{\partial}{\partial x}[T_A C_{\theta,\theta'}\{\frac{\partial H}{\partial x} + (\frac{\partial C_{h\theta'}}{\partial x})_{x'=x}\} + T_A(\frac{\partial C_{h\theta'}}{\partial x})] = 0$$

Now this is a <u>system of **two** equations for **two** unknowns for all x and x'</u> within the domain of interest.

Typical choice for: $C_{\theta,\theta'} = \sigma_{\theta}^2 e^{-\left|x-x'\right|/D}$