Chapter 15

M1: Advection of Salinity Plume

Contents

15.1 Problem Specification

M1 Advection of salinity plume in uniform flow.

Focus advection algorithm, downstream boundary.

Channel geometry and hydrodynamic boundary conditions are the same as schematic application H1. As hydrodynamic initial conditions, use the computed STEADY STATE prediction from H1. The longitudinal dispersion coefficient $E_x = 0$.

The contaminant initial conditions at $t = 0$ are

$$C(x,0) = 1 \exp\left[-c\left(\frac{x-x_0}{b_{\mathrm{M1}}}\right)^2\right] \quad \text{ft} \tag{15.1.1}$$

where $c = \ln 2 = 0.6931$, $x_0 = 5{,}000$ ft and $b_{\mathrm{M1}} = 1{,}500$ ft.

The contaminant open boundary conditions are unconstrained outflow.

Use a fixed computational time step $\Delta t = 120$ s and a fixed computational space step of Δx = 500 ft.

Compute and write to file in the STANDARD FORMAT the initial conditions at $t = 0$ and the model predictions for every time step to $t = 30\Delta t$ s.

15.2 Background

The contaminant transport Equation 2.4.4 is a second order, parabolic partial differential equation. Without either dispersion or contaminant sources however, Equation 2.4.4 becomes just

$$\underbrace{\frac{\partial}{\partial t}(AC)}_{\substack{\text{contaminant}\\ \text{storage}}} + \underbrace{\frac{\partial}{\partial x}(QC)}_{\substack{\text{contaminant}\\ \text{advection}}} = 0 \tag{15.2.1}$$

which is a first order hyperbolic partial differential equation. Numerical solutions of this deceptively simple looking equation are notoriously difficult (Sobey 1984). Numerical algorithms formulated on a fixed Eulerian grid are prone to serious numerical dispersion and solution oscillation errors. What is expected and what is numerically achieved is often quite different.

What is expected is directly available from a constant Q and constant A form of Equation 15.2.1,

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = 0 \tag{15.2.2}$$

where $U = Q/A$. This is equivalent to the ordinary differential equation system

$$\frac{dC}{dt} = 0 \text{ along } \frac{dx}{dt} = U \tag{15.2.3}$$

and $C(x,t)$ is unchanged along the particle paths $dx/dt = U$, and the initial conditions are advected with the flow without any change of form. With Equation 15.1.1 as initial conditions, the analytical solution is shown in Figure 15.1a.

15.3 Contra Costa Water District

No response[1].

[1] The following commentary was provided by CCW (Shum, 27 April 2001): "CCWD was unable to respond to any of the mass transport problems because of the effort required and a lack of justification for this effort."

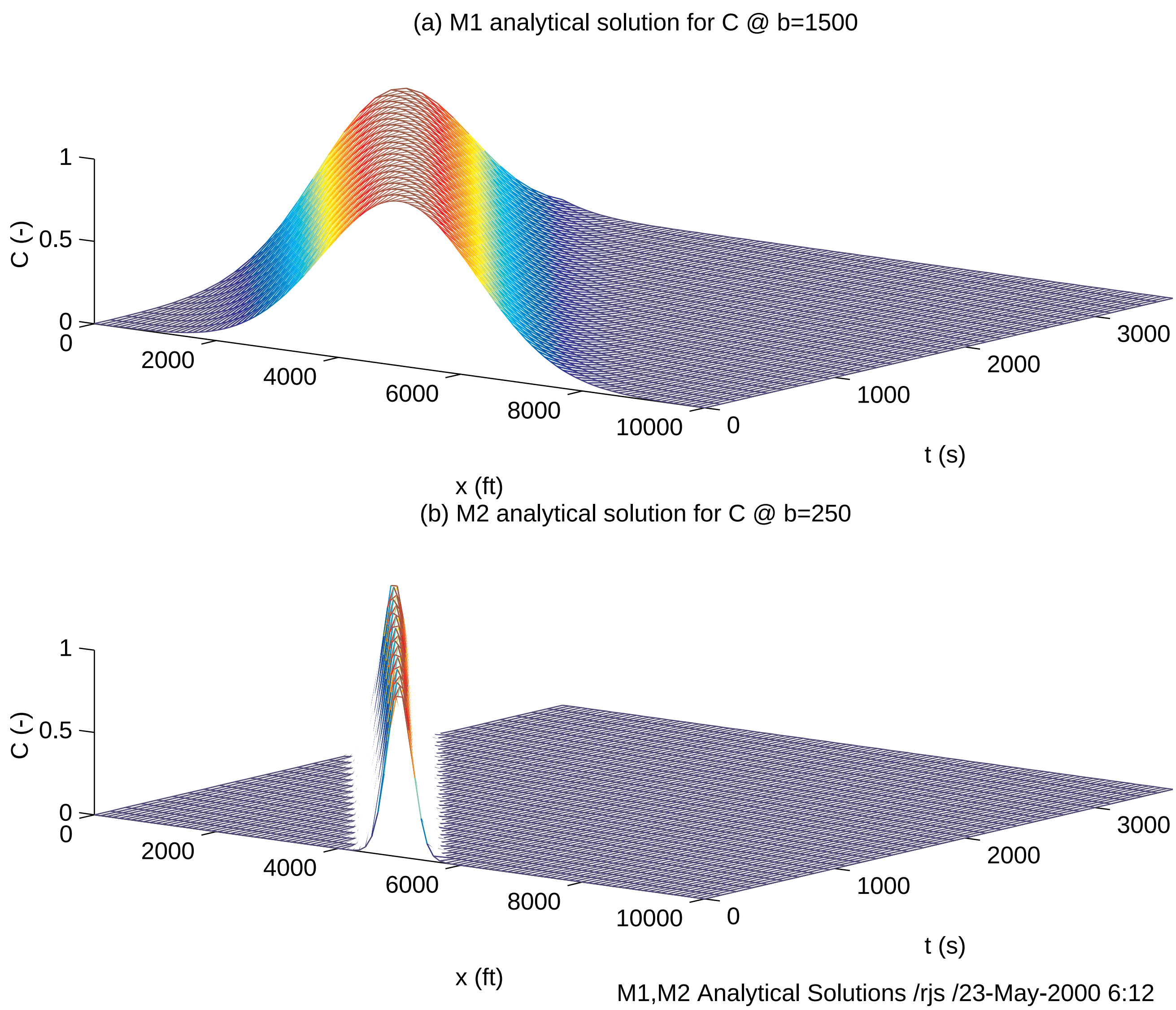

Figure 15.1: Analytical Solutions for Contaminant Advection.

15.4 Department of Water Resources

Figure 15.2a shows the DWR-predicted[2] evolution of the salinity plume. Figure 15.2b shows the error field, computed as $C_{\text{numerical}}(x,t) - C_{\text{analytical}}(x,t)$. Ideally, part (a) should be identical to Figure 15.1a and part (b) should be zero throughout. The Figure 15.2a numerical model prediction is not identical to the Figure 15.1a analytical solution. But the differences are a direct consequence of the DWR re-definition of problem. The plume is advected in the positive x direction in Figure 15.2a, but in the negative x direction in 15.1a. In addition, data was reported only every 900 s, rather than 120 s, so that the data set is very sparse in the t direction. The actual DWR predictions are indicated by the circle markers. The balance of the surface plot is the standard response of surface or contouring, which interpret the sparse t data as isolated ridges. The circle markers suggest excellent algorithm performance. This is confirmed by the error field plot in Figure 15.2b where the maximum error is of order 4×10^{-4} and insignificant. The expected excellent performance of the DWR model (Jobson and Schoellhamer 1987) for pure advection has been demonstrated.

Figure 15.2c shows the DWR-predicted contaminant mass balance. Unfortunately, the poor t resolution makes it difficult to be certain if this or is not an acceptable result.

[2]The DWR data file has the predicted solution reported only every 900s, instead of the required 120 s, the x axis directed in the wrong direction, and the time and reach number listed in inverted order. Appropriate corrections have been made for the following analyses.

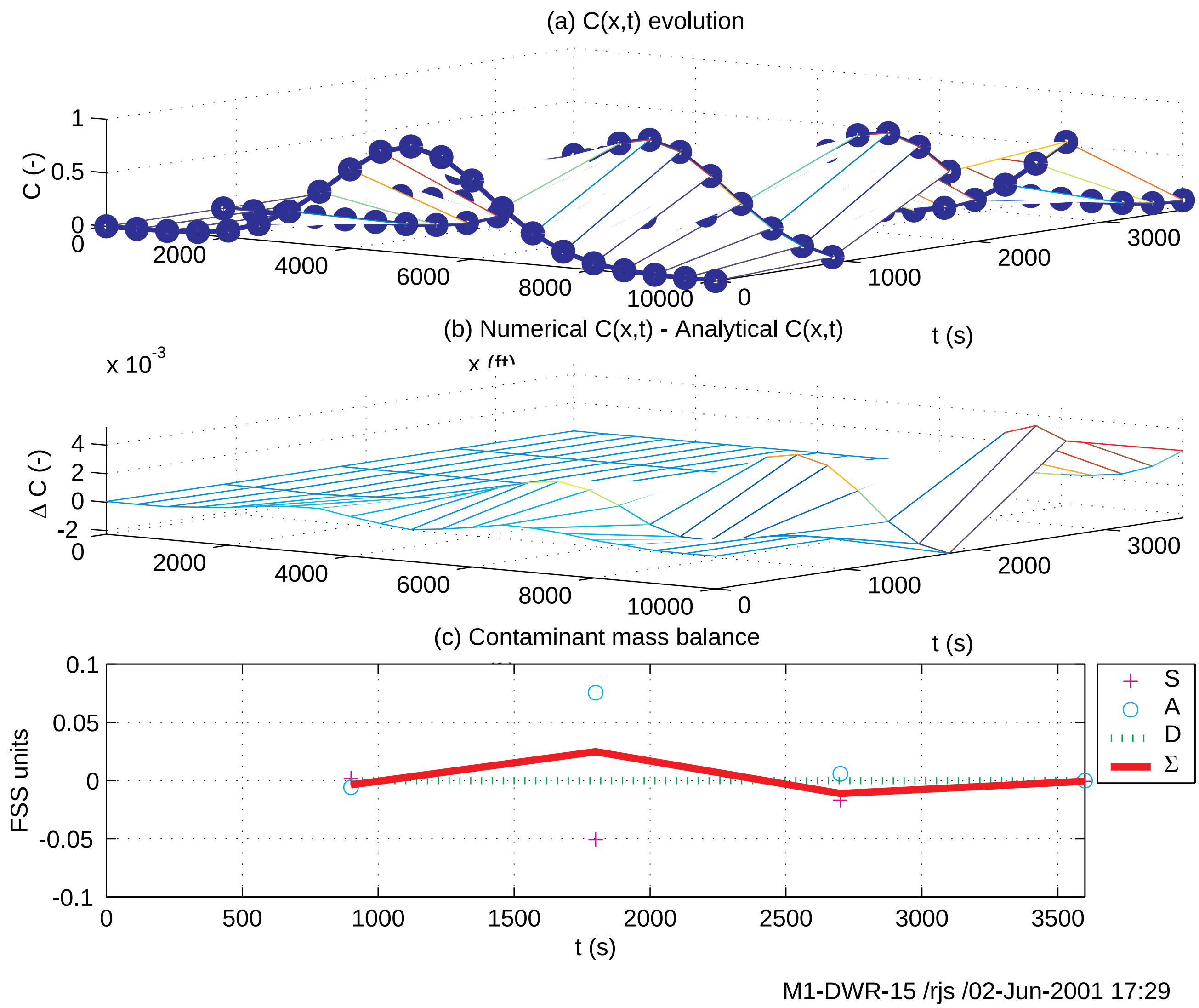

Figure 15.2: M1 DWR-predicted Advection of Salinity Plume, Error Field and Contaminant Mass Balance.

15.5 Resource Management Associates

Figure 15.3a shows the RMA-predicted evolution of the salinity plume. Figure 15.3b shows the error field, computed as $C_{\text{numerical}}(x,t) - C_{\text{analytical}}(x,t)$. Ideally, part (a) should be identical to Figure 15.1a and part (b) should be zero throughout. Overall, this is the expected result. Figure 15.3a reasonably follows Figure 15.1a. The error field, Figure 15.3b, shows evidence of numerical dispersion and solution oscillations, but they are within reasonable bounds.

Figure 15.3c shows the RMA-predicted contaminant mass balance. There is a minor oscillation in the contaminant mass balance, presumably a consequence of the numerical dispersion and solution oscillations identified in Figure 15.3b.

REFERENCES

Jobson, H. E. and D. H. Schoellhamer (1987). Users manual for a branched Lagrangian transport model. Water-Resources Investigations Report 87-4163, U.S. Geological Survey, Reston, VA.

Sobey, R. J. (1984). Numerical alternatives in transient stream response. *Journal of Hydraulic Engineering 110*, 749–772.

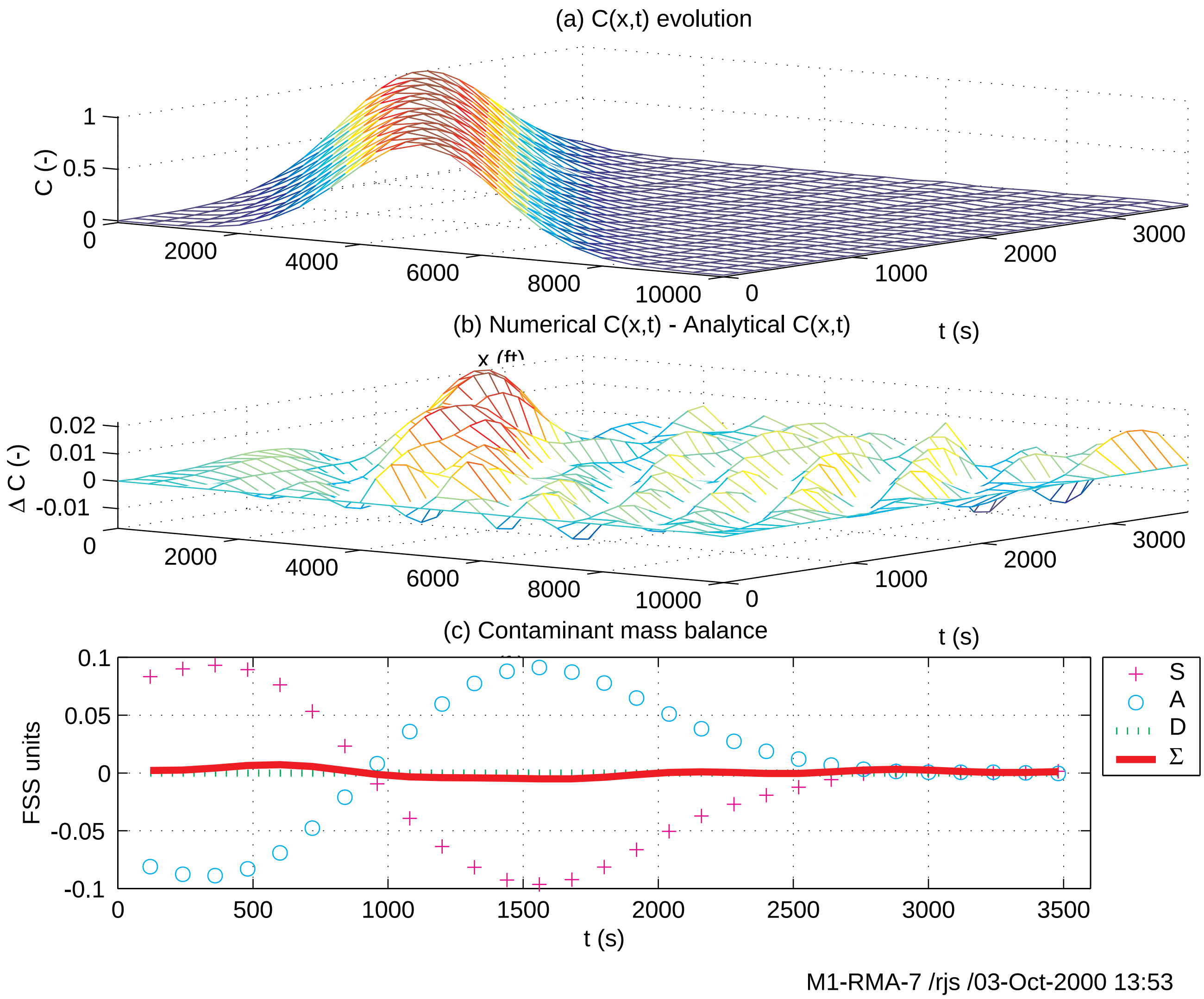

Figure 15.3: M1 RMA-predicted Advection of Salinity Plume, Error Field and Contaminant Mass Balance.